DETERMINATION OF THE LINEAR BEDROCK COEFFICIENT FROM HISTORICAL FLOW DATA

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Background:

In order to predict stream or river flow from outputs generated by the Cornell Soil Moisture Routing (CSMR) model, a bedrock or base flow component must be determined. Currently, the CSMR predicts percolation flow for each cell within a given basin. This percolation flow exits the shallow subsurface and enters a bedrock aquifer which is assumed to be linear in nature; that is the outflow from the bedrock aquifer in the form or springs and seeps is linearly dependent on the storage in the aquifer:

$$Q_b = \alpha S$$

where α is a constant with units of one over time. As the storage in the aquifer increases, the outflow will increase and conversely as the storage in the aquifer decreases the outflow from the aquifer will decrease.

It is of paramount importance to quantify and justify values used in the CSMR model. A recent reviewer of Dr. J. Frankenberger's paper, ***A GIS-based Variable Source Area Model for Shallow Sloping Soils***, questioned the robustness of using the lumped linear reservoir model to predict base flow in streams (see Reviewer#3's Page 4, Paragraph 4 comment). Due to lack of data included in the aforementioned paper, this brief statistical analysis was performed to determine a general bedrock reservoir coefficient for the greater Delaware River Watershed.

Theory:

The mass balance equation for the linear reservoir can be written as follows:

$$\frac{dS}{dt} = Q_i - Q_o$$

where S is the storage, Q_i is the inflow flux, and Q_o is the outflow flux. If we choose a seasonal period for analysis when the inflow flux may be assumed negligible (either winter or summer when low moisture contents prevent downward percolation flow), then our mass balance equation becomes:

$$\frac{dS}{dt} = -Q_o = -Q_b$$

where Q_b is the base flow in the stream or river. Substituting our equation for Q_b into our mass balance equation yields:

$$\frac{dQ_b}{dt} = -\alpha Q_b$$

Separating variables and integrating with respect to time yields the common streamflow recession equation:

$$\ln Q_b = \ln Q_{b_o} - \alpha(t - t_o)$$

which has the functional form:

$$\ln Q_b = mt + C$$

where C is a constant and m is the slope of the regression line of $\ln(Q_b)$ versus time. Determination of m, of course, yields our linear bedrock reservoir coefficient. Thus by plotting the logarithm of stream flow versus time and regressing "linear" segments of record, a sampling of bedrock coefficients can be determined.

Methodology

Three historical rivers were chosen for this study, the West Delaware at Walton, NY, Mill Brook, and Tremper Kill. Drainage area for each river was 860 km^2, 86 km^2, and 65 km^2, respectively. Daily peak stream flow records were used for the regression analysis. The logarithm of flows for summer (June through September) and winter (December through February) were plotted versus time to find "linear" segments. Best fit lines were then determined for each linear segment resulting in a sampling of bedrock coefficients.

Obviously when analyzing the recession limbs of river hydrographs, some subjectivity is introduced simply in the choice of events and the fit of the lines. Thus for the ten years analyzed (for a total of twenty seasons), some arbitrary guidelines were chosen that seemed reasonable:

(1) Seven consecutive receding flow values were needed for each analysis.
(2) The first receding flow value was truncated from the series so as to attempt to eliminate any influence of inflow to the bedrock reservoir.
(3) A linear r^2 fit of at least 0.85 was necessary for the specific reservoir coefficient to be included in the statistical analysis.

After reservoir coefficients were determined, a statistical analysis was done both on their values and their r^2 values to determine mean reservoir coefficients and also to give a measure of how well these values fit the data.

Results and Discussion

In total 168 recession events were analyzed. Figure 1 presents the histogram of the aquifer coefficients while Figure 2 shows the histogram of the r^2 values. The mean and median values for each distribution are very similar, thus either could be used for a representative value of bedrock coefficient. As shown in Figure 1, the mean bedrock coefficient is 0.096 day^{-1}, with a standard deviation of 0.039 day^{-1}, yielding a coefficient of variation equal to 40%. The bedrock coefficient of 0.096 agrees fairly well with that used by Dr. J. Frankenberger of 0.12 day^{-1}(20% less), when simulating stream flows.

The frequency distribution of r^2 values shows that the fit for the regression lines was very good. The mean r^2 value was 0.964 with a standard deviation of only 0.032. Obviously we forced our fit to be good in this study by setting a cutoff at 0.85. Presentation of our r^2 distribution may be essential in answering Reviewer #3's question, "...how well was the fit."

Although the question of deviations in bedrock coefficient values on actual base flow predictions was not asked by the reviewers, it is a question which may arise on the second submission. Thus we ask and answer two questions here:

(1) What will be the differences in base flow predicted by Dr. J. Frankenberger's reservoir coefficient of 0.12 day^{-1} and our mean value of 0.096 day^{-1}?

(2) What will be the differences in the base flow predicted by a range in reservoir coefficient values found in our statistical analysis?

To answer (1) we simulate base flow in the Fall Clove watershed for 04/85-04/86 using percolation values supplied by the CSMR and using a bedrock coefficient value of 0.12 and 0.096 day^{-1}. Figure 3 shows the results of our calculations. Figure 3 shows that the deviations in normalized base flow are very small when using the two coefficients and the base flow lines coincide and cross multiple times over the course of a year's time. To answer (2) we again simulated the Fall Clove watershed base flow using our mean value and our mean value plus and minus one standard deviation. Figure 4 shows the results of this analysis. Although the deviations seen in this graph are larger than those seen in Figure 3, they are still small when compared to the magnitudes of flows typically seen in the watersheds.

Comments and Conclusions

I believe from the results of this study that we may statistically present the average bedrock coefficient as that which was used by Dr. J. Frankenberger in her study. Although the reviewers may question the variation in the coefficients, we may be able to appease them by a figure similar to Figure 4. As a side note, the variability in the coefficients indicate that the bedrock reservoir may not truly be linear, but in our approximation it works well enough.

Frequency Distribution of Linear Bedrock Aquifer Coefficients

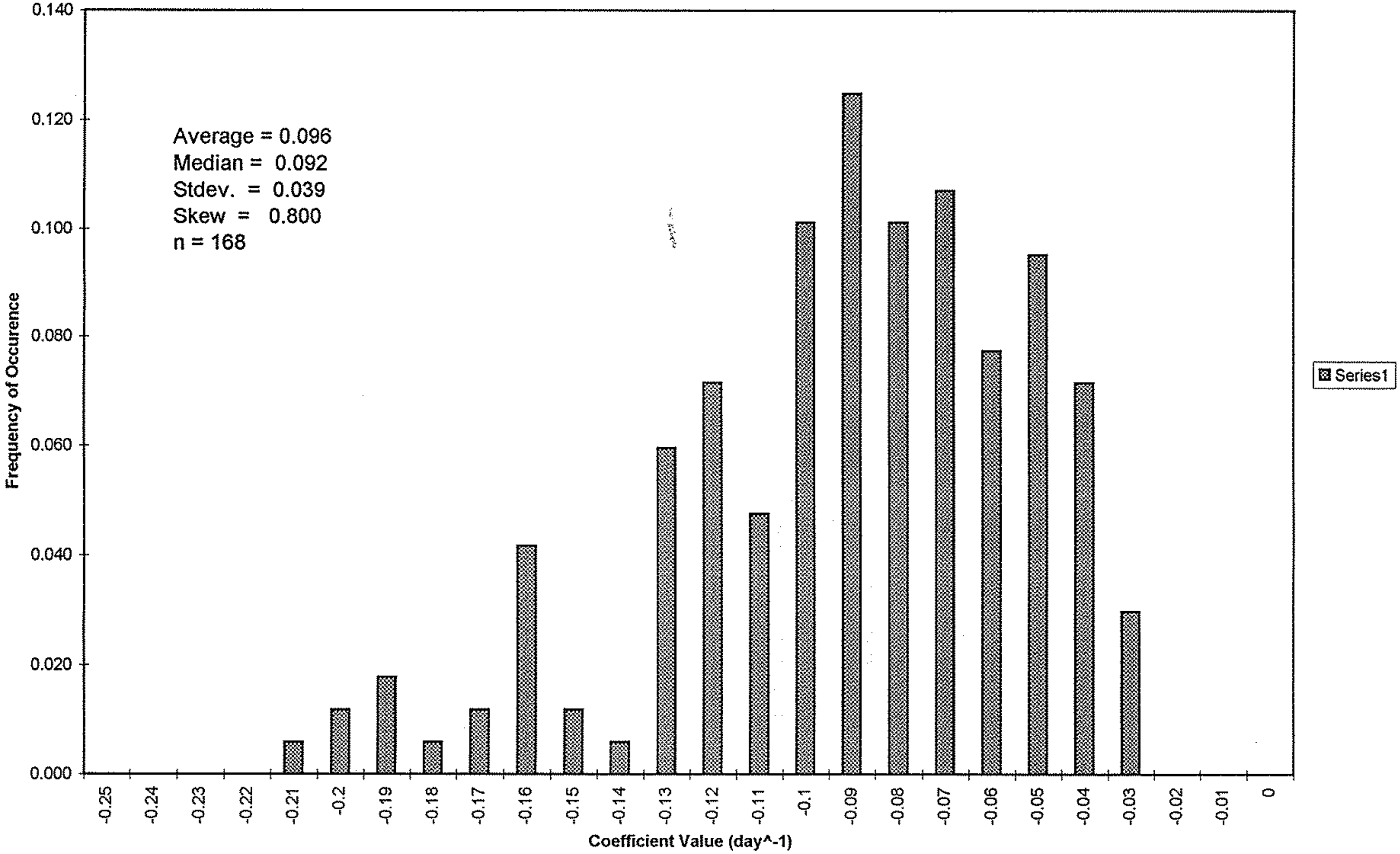

Fig. 1

Statistics Chart 5

Histogram of R^2 Values for Log-Linear Regression

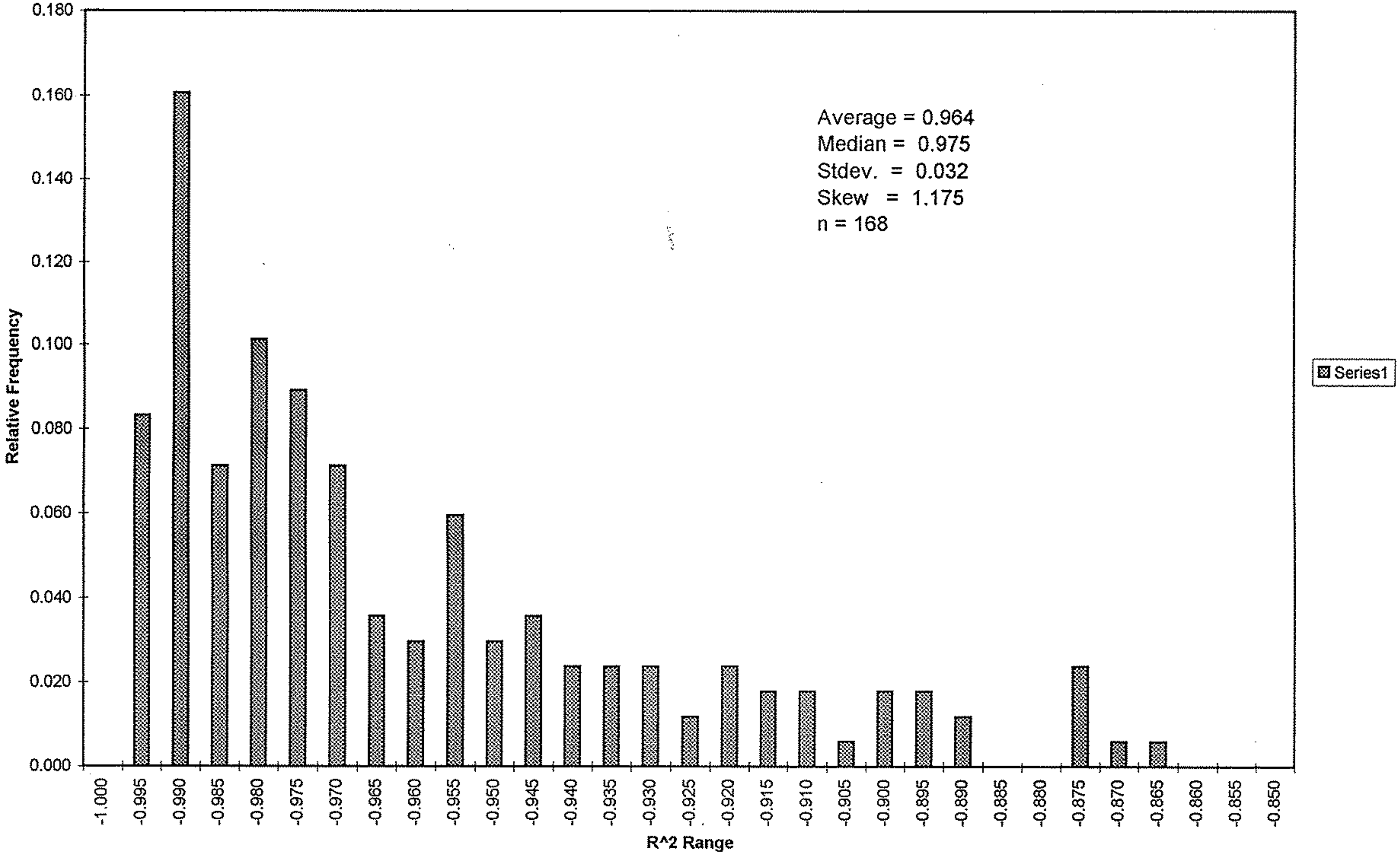

Fig. 2

Comparison of Bedrock Flow Rates Using Different Bedrock Coefficients

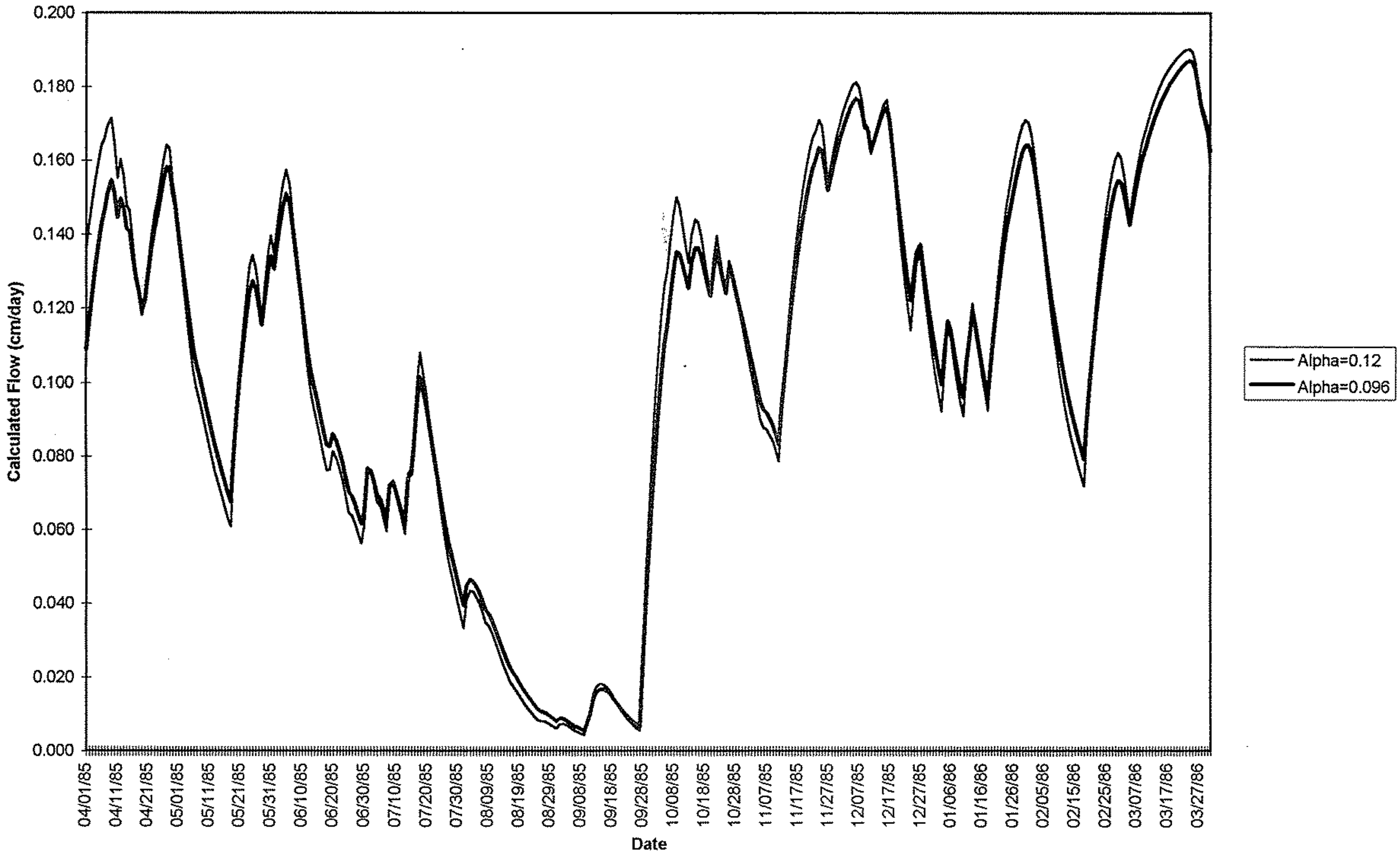

Fig. 3

Comparison of Base Flow Rates Using Mean and One Standard Deviation of Bedrock Coefficient Dist.

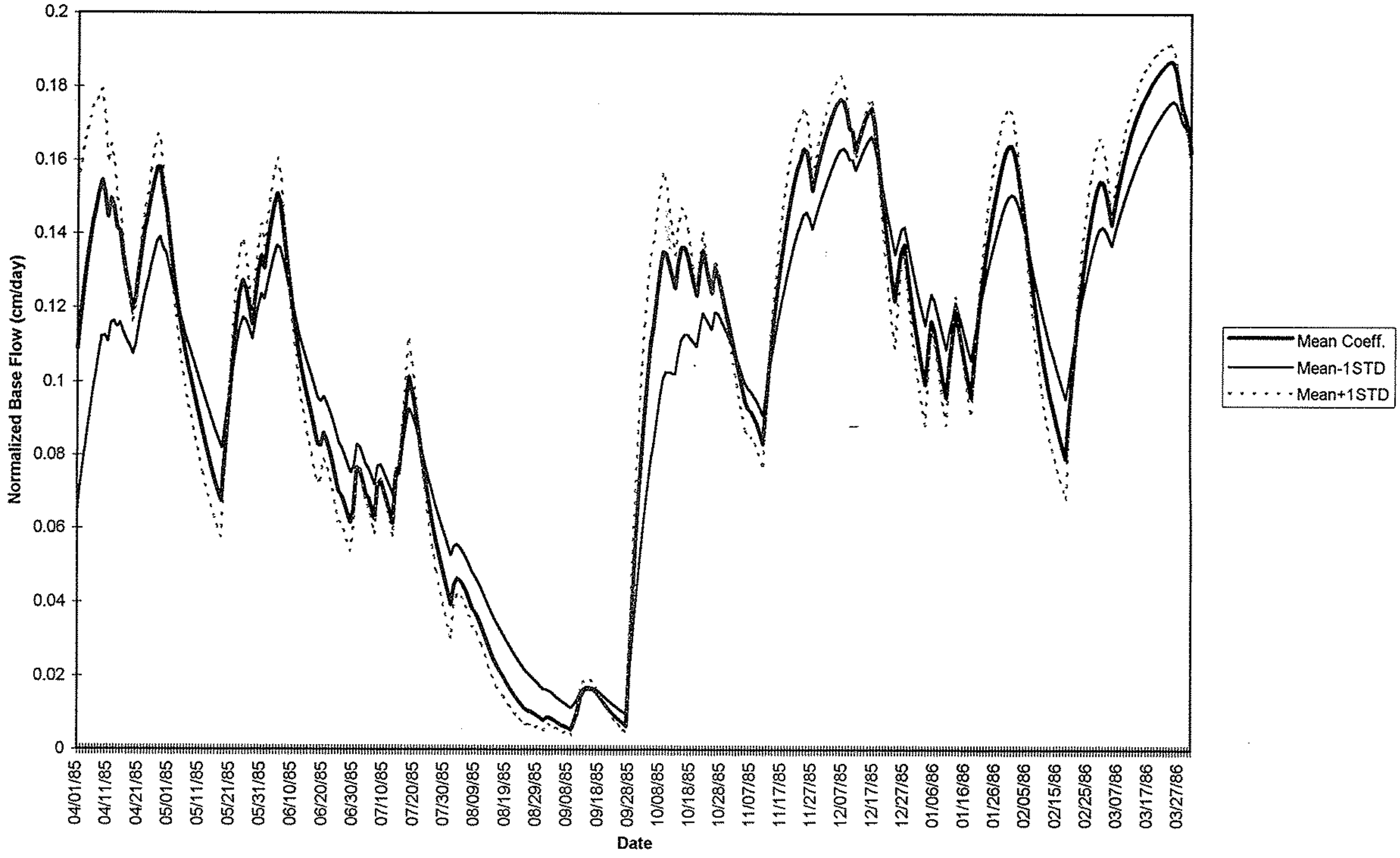

Fig. 4