

## Refined conceptualization of TOPMODEL for shallow subsurface flows

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### Abstract

The TOPMODEL framework was used to derive expressions that account for saturated and unsaturated flow through shallow soil on a hillslope. The resulting equations were the basis for a shallow-soil TOPMODEL (STOPMODEL). The common TOPMODEL theory implicitly assumes a water table below the entire watershed and this does not conceptually apply to systems hydrologically controlled by shallow interflow of perched groundwater. STOPMODEL provides an approach for extending TOPMODEL's conceptualization to apply to shallow, interflow-driven watersheds by using soil moisture deficit instead of water table depth as the state variable. Deriving STOPMODEL by using a hydraulic conductivity function that changes exponentially with soil moisture content results in equations that look very similar to those commonly associated with TOPMODEL. This alternative way of conceptualizing TOPMODEL makes the modelling approach available to researchers, planners, and engineers who work in areas where TOPMODEL was previously believed to be unsuited, such as the New York City Watershed in the Catskills region of New York State. Copyright © 2002 John Wiley & Sons, Ltd.

**Key Words** TOPMODEL; STOPMODEL; distributed hydrological model; soil moisture; interflow; shallow subsurface flow; saturation excess; variable source areas

### Introduction

The 'TOPMODEL' concept (Beven and Kirkby, 1979; Beven and Wood, 1983; O'Loughlin, 1986; Ambroise *et al.*, 1996) is currently a popular watershed modelling tool (e.g. Anderson *et al.*, 1997; Lamb *et al.*, 1998; Güntner *et al.*, 1999; Scanlon *et al.*, 2000). It is popular because of its deceptive simplicity, clever use of geomorphology, and demonstrated applicability to a wide variety of situations. Even so, various hydrologists have noted the inappropriateness of TOPMODEL's *conceptual basis* to describe, meaningfully, hydrologically shallow, hilly situations where transient, perched groundwater flow plays substantial roles in controlling watershed hydrology (Moore and Thompson, 1996; Woods *et al.*, 1997; Frankenberger *et al.*, 1999; Scanlon *et al.*, 2000). Given the large, global distribution of such systems and the ubiquity of TOPMODEL's use, it is interesting that the model has been so successful (Anderson *et al.*, 1997). Even more to the point, many published studies clearly apply TOPMODEL to shallow, perched interflow-driven systems and the results are often good (Hornberger *et al.*, 1985; Ambroise *et al.*, 1996;

Moore and Thompson, 1996; Güntner *et al.*, 1999). Scanlon *et al.* (2000) explicitly modified TOPMODEL to incorporate shallow, lateral subsurface flow by simultaneously using two TOPMODELS in parallel, one describing deep baseflow and the other describing shallow interflow. Using this modified TOPMODEL, Scanlon *et al.* (2000) succeeded in improving the model's optimization by 12.5%, which arguably suggests that the original TOPMODEL performed nearly as well. Although the Scanlon *et al.* (2000) study suggested one approach to applying the TOPMODEL concept to systems with multiple, distinct temporal flow regimes, it was unclear that it addressed the appropriateness of TOPMODEL's *conceptual basis* to shallow systems. In particular, because TOPMODEL uses 'water table depth' to describe the storage deficit, it is unclear how it applies to systems dominated by shallowly perched groundwater for which many parts of the watershed may never or only rarely have a water table. Woods *et al.* (1997) developed a TOPMODEL-like index for shallow interflow that successfully retained water table depth as the state variable by referencing it to an impermeable layer and mathematically defining the index in a way that the depth of the saturated zone approaches zero during dry conditions. Unlike Scanlon *et al.* (2000), the Woods *et al.* (1997) approach did not facilitate deep baseflow. This note presents an alternative approach for deriving a shallow-soil TOPMODEL (STOPMODEL), extending the theoretical basis for TOPMODEL by replacing saturated hydraulic conductivity as a function of depth with hydraulic conductivity as a function of soil moisture content. As in the case of Ambroise *et al.* (1996), this study leads to continued development of the TOPMODEL concepts and more general flow representations.

### Model Derivation

In lieu of using water table depth  $z$  as the state variable, depth-averaged soil moisture is more convenient when considering shallow soil systems with transient perched water tables. As with the depth-transmissivity functions common to TOPMODEL, many expressions exist for hydraulic conductivity as a function of moisture content and any could be applied. We considered the hydraulic conductivity of the soil  $K(\theta)$  as a function that

declines exponentially with declining soil moisture deficit (e.g. Hillel, 1980) in the form:

$$K(\theta) = K_s \exp(-\kappa\Omega)$$

$$\Omega = 1 - \frac{\theta - \theta_d}{\theta_s - \theta_d} \quad (1)$$

where  $\theta$  is the average soil moisture content,  $\theta_s$  is the saturated soil moisture content,  $\theta_d$  is the air dry soil moisture content,  $K_s$  is the saturated conductivity of the soil matrix,  $\kappa$  is a soil characteristic constant, and  $\Omega$  is the reduced moisture deficit per unit volume of soil from saturation;  $\Omega$  is equal to  $1 - \Theta$ , where  $\Theta$  is the reduced moisture content:  $\Theta = \frac{(\theta - \theta_d)}{(\theta_s - \theta_d)}$ . Note, Equation (1) implicitly assumes that the moisture content can be higher than the saturated moisture content, which is similar to the common assumption in TOPMODEL, whereby the water table can extend above the ground surface (Sivapalan *et al.*, 1987).

Using the commonly employed kinematic approximation, whereby the hydraulic gradient is equal to the topographic slope, the flux per unit width of topographic contour line  $q^i$  at location  $i$  can be obtained assuming an average water content with depth as:

$$q^i = D^i K_s^i \tan \beta^i \exp(-\kappa\Omega^i) \quad (2)$$

where  $D^i$  is the local depth to the hardpan, bedrock, or other type of restrictive layer and  $\beta^i$  is the slope at location  $i$ . Under quasi-steady-state conditions with spatially uniform recharge  $R$ , the flux  $q^i = a^i R$  and Equation (2) can be rewritten as:

$$a^i R = T_s^i \tan \beta^i \exp(-\kappa\Omega^i) \quad (3)$$

where  $a^i$  is the upland drainage area contributing to point  $i$  per unit contour length and  $T_s^i$  is the transmissivity of the soil profile at point  $i$ , i.e. the product of the saturated conductivity  $K_s$  and the depth to the restrictive layer  $D$ . Because infiltrating precipitation is closely coupled with shallow, perched groundwater,  $R$  is taken as excess precipitation (precipitation minus evaporation), similar to the approach used by Frankenberger *et al.* (1999). It may be desirable to modify  $R$  to reflect water losses to deep percolation or interception. Rearranging Equation (3) and defining the soil-topographic index similarly to Beven (1986)

and Ambroise *et al.* (1996) as  $\zeta^i = a^i / (T_s^i \tan \beta^i)$  gives the following expression:

$$\Omega^i = -\frac{1}{\kappa} \ln(\zeta^i R) \quad (4)$$

Integrating Equation (4) over the area  $A$  of the whole watershed area gives the spatially averaged soil moisture deficit:

$$\bar{\Omega} = -\frac{1}{A} \int_A \frac{1}{\kappa} \ln(\zeta^i R) dA \quad (5)$$

As with TOPMODEL, the STOPMODEL recharge term can be eliminated by combining Equations (4) and (5) (Ambroise *et al.*, 1996):

$$\begin{aligned} \Omega^i - \bar{\Omega} &= -\frac{1}{\kappa} (\ln \zeta^i - \gamma) \\ \gamma &= \frac{1}{A} \int_A \ln \zeta^i dA \end{aligned} \quad (6)$$

where  $\gamma$  has the same definition as the spatial average of the soil-topographic index used in TOPMODEL (Beven and Kirkby, 1979). Following the notation by Ambroise *et al.* (1996),  $\kappa\Omega$  can be replaced with  $\delta_S$ , where  $\delta_S$  is the relative water storage deficit for STOPMODEL, *viz.*:

$$\delta_S^i - \bar{\delta}_S = -(\ln \zeta^i - \gamma) \quad (7)$$

where  $\delta_S$  is the basin average relative water storage deficit. Equation (7) is the same expression that is used in TOPMODEL for the relative storage deficit (Ambroise *et al.*, 1996). Recall that for TOPMODEL the relative storage deficit is  $\delta_T^i = z^i / m$ , where  $z^i$  is the variable water table depth at  $i$  and  $m$  is a scaling factor.

Following Ambroise *et al.* (1996), we can write the shallow subsurface flow out of the basin  $Q_b$  as:

$$\begin{aligned} Q_b &= Q_0 \exp(-\bar{\delta}_S) \\ Q_0 &= A \exp(-\gamma) \end{aligned} \quad (8)$$

where  $Q_0$  is typically defined as a model outflow parameter related to soil hydraulic properties and topography and calibrated with  $m$ ;  $\kappa$  replaces  $m$  in STOPMODEL.

In TOPMODEL, Equation (8) is used to describe the baseflow from the saturated groundwater zone,

and additional algorithms for root zone and vertical drainage processes are required to build a complete, continuous hydrological model. In the case of STOPMODEL, Equation (8) describes flow from above a shallow restrictive layer and additional algorithms are needed to describe flow through the restrictive layer (i.e. leakage) and deep groundwater flow  $Q_{gw}$ . Modelling approaches for these additional processes are well documented in the literature and will not be discussed here. In keeping with the TOPMODEL modus operandi, details of how to model these processes can be added to the TOPMODEL collection of modelling tools. Frankenberger *et al.* (1999) and Scanlon *et al.* (2000) each suggest plausible, and different, approaches for modelling these flows.

## Discussion and Implications

Despite conceptual differences between TOPMODEL and STOPMODEL in representing flow in the landscape, the mathematical expressions for flow retain the same shape by simply replacing TOPMODEL's saturated hydraulic conductivity function with respect to water table depth  $z$  with a hydraulic conductivity function that uses soil moisture deficit  $\Omega$ , i.e. STOPMODEL. Because the basic mathematical expressions for TOPMODEL and STOPMODEL are so similar, it is probable that the performance of the models would be similar, especially when calibrated to the same streamflow hydrographs. This similarity helps explain why TOPMODEL often works well in shallow, perched interflow systems, where water tables rarely, or never, underlie the whole watershed. (e.g. Hornberger *et al.*, 1985; Ambroise *et al.*, 1996; Moore and Thompson, 1996; Güntner *et al.*, 1999; Scanlon *et al.*, 2000). It is well established that different models and input data sets can predict the outflow hydrograph equally well (e.g. Franchini and Pacciani, 1991). This was strikingly shown for the Coweeta experiment (Hewlett and Hibbert, 1963), where Boussinesq's equation (saturated flow) and Richard's equation (unsaturated flow) predicted the same outflow hydrograph (Steenhuis *et al.*, 1999).

TOPMODEL and STOPMODEL, like all hydrological models, are abstractions of the reality and, therefore, will not capture all the behaviours of a watershed. In fact, it is reasonable to assume that the processes addressed by STOPMODEL and TOPMODEL probably operate simultaneously in many

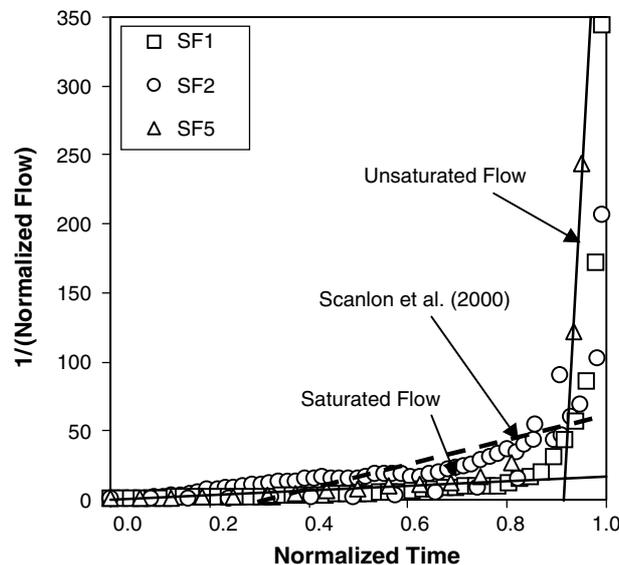


Figure 1. Normalized flow and time relationships for three storm flow episodes, SF1, SF2, and SF3, based on data from Scanlon *et al.* (2000). The dashed line shows the saturated flow relationship suggested by Scanlon *et al.* (2000) and the solid lines show the relationships that result from assuming two flow regimes, perhaps saturated and unsaturated as labelled

systems. Avoiding TOPMODEL's use of a water table depth leads to the introduction of some new conceptual issues in STOPMODEL. For example, at first glance one might interpret Equation (1) as implying that no saturated flow occurs until the entire profile is saturated. Like TOPMODEL, STOPMODEL's explicit physical interpretation of the modelled system is relaxed during calibration and parameters are optimized independently of the actual distributed physical properties. Boll *et al.* (1998), Frankenberger *et al.* (1999), and Brooks *et al.* (2000) used similar conductivity functions in their watershed models and suggest that a soil profile with an average soil moisture content above 'field capacity' can be visualized as having two moisture regimes: one at 'field capacity' and one at saturation. Therefore, a physically consistent explanation for Equation (1) is that the effective conductivity for average moisture contents above 'field capacity' reflects depth-averaged flow from the unsaturated and saturated regions. One advantage of using Equation (1) over the traditional TOPMODEL functions is that  $K_s$  can be chosen or calibrated to reflect preferential flow at high moisture contents.

Although admittedly speculative, STOPMODEL's use of a soil-moisture-dependent hydraulic conductivity function may be useful in capturing some of the curious results of Scanlon *et al.* (2000) for a small

well-monitored watershed in Virginia, where both the shallow, perched water table and streamflow were continuously measured. Scanlon *et al.* (2000) showed that, during one mid-sized storm (Storm B in Scanlon *et al.* (2000)), none of the piezometers (six sites at three depths per site) recorded the presence of a water table, yet the stream showed a sharp increase in flow. One can hypothesize several explanations, including the plausible possibility that unsaturated flow contributed substantially. Additional evidence of unsaturated flow could be interpreted from Scanlon *et al.*'s (2000) relationships between normalized shallow storm discharge (inverse) and normalized time (Figure 1). Figure 1 was created from the published data of Scanlon *et al.* (2000) and clearly shows that there are two distinct portions to the curve: an early flat part and a late steep part. Scanlon *et al.* (2000) presented three such figures (figure 9a–c in the original published paper) that all showed similar systematic trends and perhaps suggested two distinct flow regimes. One explanation is that the early flow regime is primarily saturated flow and the later flow regime is controlled by unsaturated flow, as labelled in Figure 1.

Obviously, the implementation of STOPMODEL may differ somewhat from TOPMODEL because of the conceptualized locations where the water

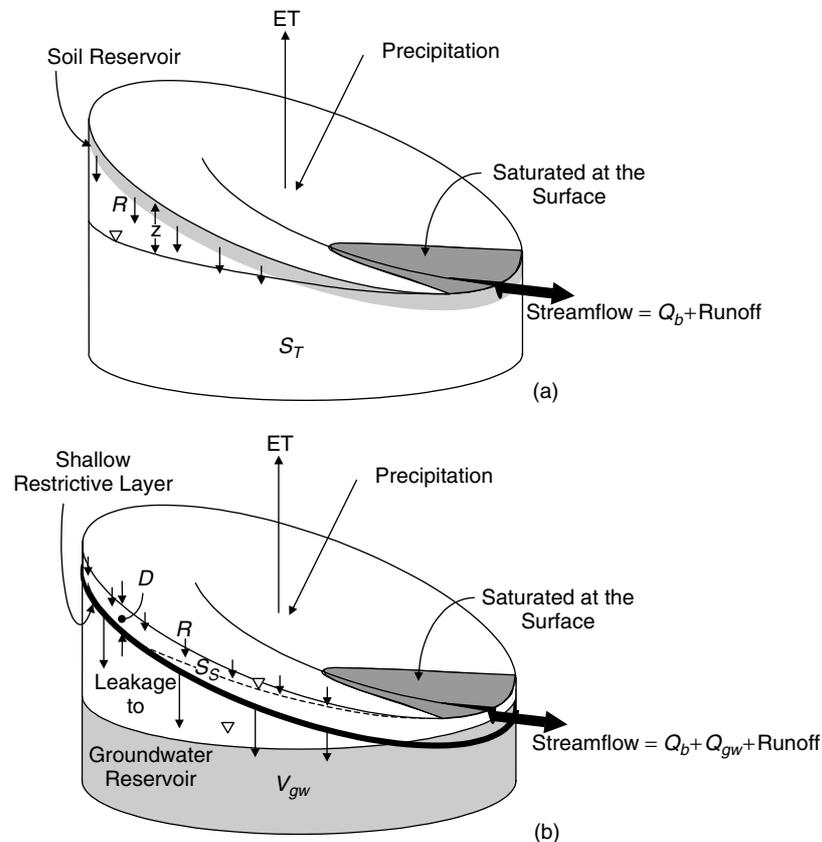


Figure 2. Schematics of the conceptual design for TOPMODEL (a) and STOPMODEL (b).  $S_T$  and  $S_S$  are the primary water storage reservoirs for TOPMODEL and STOPMODEL respectively.  $V_{gw}$  is STOPMODEL's deep groundwater reservoir storage and  $Q_{gw}$  is the outflow to the stream from this reservoir

that controls variable source area hydrology is stored (Figure 2). TOPMODEL assumes the water is recharged from the root zone into a deeper reservoir (Figure 2) from which evapotranspiration (ET) is relatively small. STOPMODEL, on the other hand, assumes the water resides in or near the root zone above a shallow restrictive layer (Figure 2) and is subject to substantial ET losses. One implication of this difference is that stream discharge and the moisture distribution within the watershed modelled by STOPMODEL and TOPMODEL may be substantially different during periods of high ET relative to recharge. For a landscape in Australia, Grayson *et al.* (2001) observed that TOPMODEL functioned well during wet periods but incorrectly predicted moisture distribution during dry periods. This may indicate that the primary hydrological processes during dry

periods are relatively independent of topography. Thus, perhaps a STOPMODEL approach would capture this phenomenon, because STOPMODEL and TOPMODEL will probably behave similarly during wet periods of the year but STOPMODEL allows the deep groundwater reservoir to be modelled in a way that is independent of topography (Figure 2).

## Conclusion

TOPMODEL was re-conceptualized for watersheds that are driven by shallow, transient, perched groundwater without substantively changing the governing equations. The result was STOPMODEL. This re-conceptualization of the popular TOPMODEL approach helps explain why TOPMODEL often works well in systems that cannot be easily characterized by a continuous, underlying water table.



Because the resulting flow equations are very similar and both models are calibrated to streamflow, it is probable that TOPMODEL and STOPMODEL will behave very similarly, especially during wet periods of the year. However, visualizing watershed hydrology from the perspective of STOPMODEL may suggest new ways of applying the TOPMODEL concept to some systems and situations where TOPMODEL has not worked well.

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