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Reply

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Michel [1999] makes an interesting contribution to our paper, which helps clarify our point of view, and we are grateful for his comment.

As noted by *Michel* [1999], *Steenhuis et al*. [1999] show that Boussinesq's and Richards' equations, when appropriately simplified, yield one and the same analytical description of a flow through a sloping shallow soil hillside. *Michel* [1999] chose another relation between the cumulative drainage and time which fits the data very well in Figure 1 except for short times. This confirms our main point, as one of the objectives of our paper was that physically different processes (Boussinesq and Richards in our case) can yield the same outflow pattern. Hence it is unsafe to assume that agreement with outflow data is a reliable criterion to justify a particular process. As we will show later, one can invent models that agree with the outflow data just as well.

Michel [1999] continues, "The discrepancy between predicted and actual hydrographs is wide and cannot easily be explained without rejecting the Darcy law." Indeed, the cause of discrepancy was suggested by us, and earlier authors, to be possibly linked to preferential/macropore flow. Actually, we thought our elementary results to be "in surprisingly good agreement with experimental results." However, again, the point of the paper was not so much the accuracy of the results but rather that one cannot distinguish clearly the differences between models based on Boussinesq's and Richards' equations and possibly others like Michel's.

If we take our (14) for the cumulative drainage $M(t)$, it can be rewritten as

$$M(t) = 2M_\infty \frac{t}{t^*}\left(1 - \frac{t}{2t^*}\right), \qquad t \le t^* \tag{1}$$

where M_∞ is the maximum value of M which is reached at $t = t^*$. *Michel* [1999] uses the equally simple expression

$$M(t) = M_\infty \left[\frac{t}{t+\tau}\right]. \tag{2}$$

The constants in (1) and (2) should satisfy the imposed drainage rate at time $t = 0$. Differentiation of (1) and setting $t = 0$ gives

$$\frac{dM}{dt}(t=0) = 2\,\frac{M_\infty}{t^*}. \tag{3}$$

By substituting the observed cumulative outflow $M_\infty = 1.26$ m^3 and the observed $dM/dt(t = 0) = 0.0264$ m^3/h, we find that $t^* = 96$ hours. These values, by construction, mean that (1) fits the data at the beginning and end of the measurements. They overpredict $M(t)$ at intermediary times (see Fig. 2); this is the expected effect of macropore flow. The effect of preferential paths is taken into account in the determination of t^*, then at intermediary times when those paths empty, $M(t)$ will be less than the prediction which assumes they still contribute to drainage.

Similarly, if we calculate τ in (2) so that it also satisfies the imposed initial drainage, then

$$\tau = t^*/2 \text{ (48 hours)}. \tag{4}$$

Michel [1999] took $\tau = 32$ hours, a very different value, to curve fit data at intermediary times (Fig 1); hence, by construction, it will fit the data better than (1) at those times but predicts an initial flux of 0.0394 m^3/h instead of the actual 0.0264 m^3/h. Figure 2 shows the prediction of (2) with $\tau = 48$ hours, which now satisfies the initial flux condition and becomes consistent with (1). Obviously, (2) now underestimates $M(t)$ at intermediary times. The overestimate of (1) was physically clear, it is not as obvious to us if the underestimate of (2) is equally clear physically.

It is acutally amusing that the geometric average of (1) and (2)

$$M^2(t) = 2M_\infty^2 \frac{t}{t^*} \frac{1 - \dfrac{t}{2t^*}}{1 + \dfrac{t^*}{2t}} \tag{5}$$

fits the data for all times with greater accuracy, but, of course, it should not be concluded that (5) is physically correct.

In conclusion, *Michel's* [1999] comment is very interesting because it shows that not only Boussinesq's and Richards' equations (with appropriate simplifications) lead to the same analytical result but also that other rational models can lead approximately to the same result. Furthermore, the comment confirms that outflow data should not be used for validation of models because with proper fitting of the parameters most models can predict the outflow equally well.

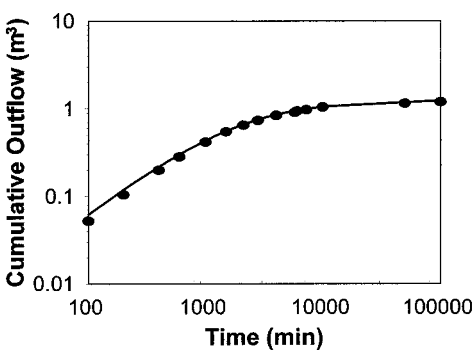

Figure 1. Actual (dots) and predicted cumulative outflow for the Coweeta experiment. The solid curve is the predicted cumulative outflow for the model of *Michel* [1999] (equation 2) with $\tau = 32$ hours.

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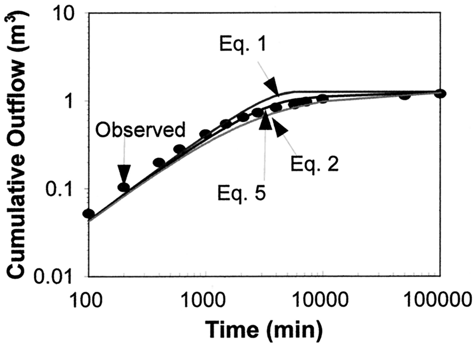

Figure 2. Actual (dots) and predicted cumulative outflow for the Coweeta experiment. The top curve is the predicted cumulative outflow from Richards' equation or the simplified Boussinesq model (Equation 1); the lowest curve is the same as in Figure 1 but with $\tau = 48$ hours; and the middle line is the geometric mean (Equation 5).

References

Michel, C., Comment on "Can we distinguish Richards' and Boussinesq's equations for hillslopes?: The Coweeta experiment revisited, *Water Resour. Res.*, this issue.

Steenhuis, T. S., J.-Y. Parlange, W. E. Sanford, A. Heilig, F. Stagnitti, and M. F. Walter, Can we distinguish Richards' and Boussinesq's equations for hillslopes?: The Coweeta experiment revisited, *Water Resour. Res.*, 35(2), 589–593, 1999.

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