SCS RUNOFF EQUATION REVISITED FOR VARIABLE-SOURCE RUNOFF AREAS

By Tammo S. Steenhuis,[1] Michael Winchell,[2] Jane Rossing,[3]
James A. Zollweg,[4] and Michael F. Walter[5]

ABSTRACT: Simple methods for predicting runoff from watersheds are important in engineering practice, particularly in flood forecasting and water-balance calculation. In this paper, we illustrate that the often used SCS runoff curve-number approach in its most elementary form can be derived from assuming that only the saturated areas contribute to direct runoff. With this approach the initial abstraction or the amount of water required before runoff starts is equal to air-filled pore space per unit area for the most shallow soil in the watershed. Air-filled pore space throughout the year was calculated with aid of the simple water balance employing the Thornthwaite-Mather procedure for the most shallow soil in the watershed. By plotting the effective precipitation defined as the amount of precipitation minus the initial abstraction against the observed runoff for two watersheds in Australia and three in the northeastern United States we found that the SCS curve-number equation in its elementary form fitted the data well.

INTRODUCTION

In humid, well-vegetated areas with shallow soils, such as the northeastern United States, Hortonian infiltration-excess overland flow does not explain observed overland-flow patterns. On shallow soils characterized by a highly conductive topsoil underlain by a dense subsoil, and in regions where the ground-water is close to the surface, runoff is usually generated from areas that are or become saturated during the storm. These areas may also contribute most pollutants associated with overland flow to the stream, and, therefore, simple methods for calculating runoff from such regions are important. Currently, there is only one engineering method, by Boughton (1987, 1990), that calculates the saturated (or contributing) areas in watersheds. It has the disadvantage of being rather cumbersome.

The objective, therefore, is to find a simple engineering method to predict the magnitude of the area that contributes direct runoff to the streamflow during a rainstorm. In the present paper, we will show that the Soil Conservation Service (SCS) runoff equation is directly based on principles used in partial-area hydrology and can predict the contributing area in a less cumbersome way than that proposed by Boughton (1990). Also, we will more precisely define the storage factor associated with the application of the SCS method.

THEORY

The basic assumption of variable-area hydrology is based on the concepts of Hewlett and Hibbert (1967), Dunne and Black (1970a, b) and, later, Boughton (1987, 1990) and Steenhuis et al. (1984). Rain that falls on unsaturated soil infiltrates, increasing the moisture content until the soil profile becomes saturated, after which additional rainfall becomes surface runoff. Since an area either contributes or does not, under this assumption, during any short time period the fraction A_f, of the watershed that contributes runoff can be expressed mathematically as

$$A_f = \Delta Q/\Delta P \tag{1}$$

where ΔQ = incremental runoff or, more precisely, the volume of excess rainfall generated during the time period divided by the whole watershed area; and ΔP = incremental depth of precipitation during the same time period. An equation often used to predict runoff is the SCS curve-number approach. The form that is typically used is (Rallison 1980)

$$Q = \frac{(P - I_a)^2}{P + S - I_a} \tag{2}$$

where I_a = initial abstraction and is set equal to $0.2S$ in its current use. The variable S is sometimes referred to as potential maximum storage. A new parameter, the effective rainfall P_e, is equal to the amount of precipitation after the runoff starts

$$P_e = P - I_a \tag{3}$$

Eq. (2) can be rewritten in the form originally proposed by Mockus [in Rallison (1980)]

$$Q = \frac{P_e^2}{P_e + S} \tag{4}$$

Mockus [in Rallison (1980)] justified the form of the equation "on grounds that it produces rainfall-runoff curves of a type found on natural watersheds."

Because I_s is the amount of water required for runoff to start, in terms of variable-source-area hydrology, I_a is equal to the amount of water that can infiltrate before complete saturation per unit area for the soil generating the first runoff. Therefore, a more accurate way to determine the initial abstraction when variable-source processes dominate than one using $0.2S$ is to actually calculate I_a by using a water-balance model for the soil with the least available storage. This is in accordance with Dickinson and Whiteley (1970), who were among the first to show the relationship between initial moisture content and contributing area.

The contributing area, according to (1), is equal to the derivative of Q with respect to P_e. Thus, differentiating (4) with respect to P_e, the portion of the watershed contributing can be found using partial fraction decomposition

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[1] Assoc. Prof., Dept. of Agric. and Biol. Engrg., Riley-Robb Hall, Cornell Univ., Ithaca, NY 14853.

[2] NSF-REU Student, Dept. of Agric. and Biol. Engrg., Riley-Robb Hall, Cornell Univ., Ithaca, NY.

[3] Grad. Student, Dept. of Agric. and Biol. Engrg., Riley-Robb Hall, Cornell Univ., Ithaca, NY.

[4] Hydro., USDA-ARS, Pasture Res. Lab, Curtin Road, University Park, PA 16802.

[5] Prof., Dept. of Agric. and Biol. Engrg., Riley-Robb Hall, Cornell Univ., Ithaca, NY.

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$$Q = P_e - S + \frac{S^2}{P_e + S} \qquad (5)$$

Then, differentiation results in

$$A_f = 1 - \frac{S^2}{(P_e + S)^2} \qquad (6)$$

At a minimum, the calculated contributing area with (6) has to satisfy the limits when $P_e = 0$ and when P_e approaches infinity. As expected, when $P_e = 0$ the contributing area is zero, and when $P_e = \infty$ the contributing area equals 1. Eq. (6) also indicates that when P equals S, 75% of the watershed is contributing. Next, we will confirm that the earlier interpretation of S by Mockus [in Rallison (1980)] as the maximum storage is correct. This is important, because it proves that the method has a physical basis and is not simply a curve-fitting routine.

Fig. 1 shows both the runoff as predicted by the runoff equation (line 2) and its tangent (line 1) at the point where the runoff amount Q^* is generated by a precipitation quantity P_e^*. The portion of the watershed contributing is A_f^* and is equal to the slope of the tangent line. Trigonometry shows that the precipitation axis is intersected by the tangent line at

$$S^* = \frac{P_e S}{P_e + 2S} \qquad (7)$$

The tangent line can be interpreted as the runoff generated from a hypothetical "tangent" watershed, in which runoff starts after the available storage of S^* has been filled and in which the contributing area remains A_f^* throughout the storm. After precipitation of P_e^*, both watersheds have a runoff amount Q^* and the same contributing area A_f^*. Thus, in other words, the "original SCS" watershed is equivalent to (i.e., gives the same result) a "tangent" watershed at P_e^*, Q^*, in which the average storage in the contributing area is equal to S^* in (7).

Eq. (7) shows that if P_e approaches infinity (and the portion of the area contributing equals 1, then S^* equals S. Thus, in other words, S is also equal to the potential average storage in the watershed (i.e., the total amount of water that can be stored in the watershed from the time that the first overland flow occurs until it is completely saturated divided by the total area). This is in accordance with the interpretation of Mockus, as expressed in a letter to Orrin Ferris in 1964 [in Rallison (1980)], that "S is limited by either the rate of infiltration or the amount of water storage available in the soil profile, whichever gives the smaller S-value . . . . In practice, rainfall intensity was neglected" (and, thus, the rate of infiltration, too).

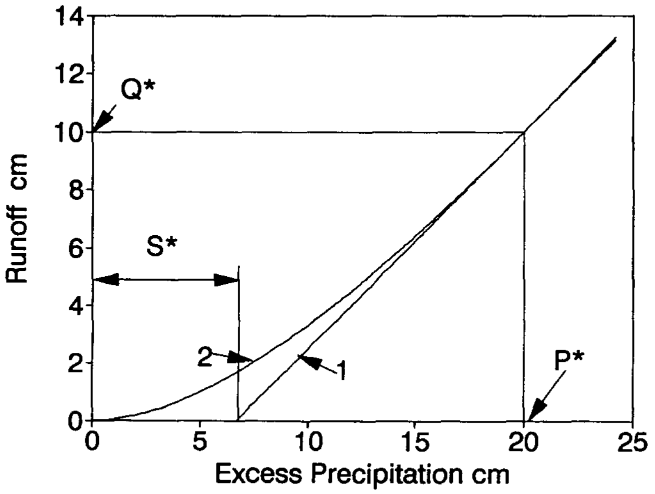

FIG. 1. Runoff as Function of Effective Precipitation for SCS Watershed and "Tangent" Watershed

As Mockus [in Rallison (1980)] and several other researchers noted, the SCS runoff equation does not contain any expression of time. One of the reasons given was the unavailability of time-dependent data. However, as we have just proven, the precipitation rates are not required to obtain runoff volume. Of course, to predict runoff rates, information about the rainfall rates is required.

We will now give a more detailed validation, using five watersheds. Two are from Australia and three from the northeastern United States.

MATERIALS AND METHODS

The runoff and precipitation data for the two watersheds (Brigalow Research Station and Back Creek) in Australia were published by Boughton (1987) for periods with low antecedent moisture conditions. The 16.8 ha Brigalow Research Station watershed is located about 400 km northwest of Brisbane and is covered by native Brigalow forest. Soils are heavy textured with some duplex profiles. The Back Creek watershed is 7,000 ha and situated 75 km south of Brisbane. Soils are derived from parent basalt material, and the original subtropical rain forest has been replaced by pasture grasses.

Proximity to a recording rain gauge was the main criterion for selecting the three watersheds in the northeastern United States. The two located in the Catskill Mountains (High Falls Brook and Biscuit Brook) were forested, with rain gauges located 5–10 km away. The other (Mahantango Creek Tributary), located in eastern Pennsylvania, was mainly agricultural and had the rain gauge on site. Table 1 summarizes the relevant data.

Data for the U.S. watersheds were obtained from several sources. For the two Catskill locations, 15-min streamflow values were obtained from the U.S. Geological Survey in Ithaca, N.Y. and Albany, N.Y. For the Pennsylvania watershed, 5-min streamflow data were obtained from the Agricultural Research Service (Gburek 1977).

Hydrographs were created from the data described for up to 40 rainfall events on a single watershed. Runoff volume was calculated by integrating the area under the hydrograph and subtracting the base flow. The base-flow separation method described by Linsley et al. (1982) was used: The surface runoff begins at the minimum point before the peak and ends at the time of recession after the peak. The times for each watershed are given in Table 2.

TABLE 1. United States Watershed Characteristics

Watershed (1)	Location (2)	Area (ha) (3)	Land use (4)	Rain-gauge location (5)
Mahantango	Pennsylvania	55	Agriculture	Obtained at site
High Falls	Catskill	715	Forest	Claryville
Biscuit	Catskill	904	Forest	Slide Mountain

TABLE 2. Hydrologic Parameters for United States Watersheds

Watershed (1)	Soil texture (2)	Total available moisture (cm) (3)	Potential evaporation in July (cm/day) (4)	Time of recession (days) (5)
Mahantango	Silt loam	21.3	0.5	0.88
High Falls	Silt loam	10.3	0.46	1.19
Biscuit	Silt loam	10.3	0.46	1.25

RESULTS

Australia

In Fig. 2, the observed runoff data are shown with the curve fitted (by eye) from the SCS equation [(4)], and the runoff data are shown as a function of the effective precipitation (amount of precipitation minus the initial abstraction) for the two Australian watersheds during the wet part of the year when the soils were at field capacity. Each data point represents different storms, some occurring over several days. The effective precipitation was obtained by subtracting the initial abstractions from the observed rainfall. The initial abstraction was measured by Boughton (1987) and amounted to 7 cm for the Brigalow watershed and 4 cm for the Back Creek watershed. The fitted lines in Fig. 2 were obtained by the initial abstraction measured by Boughton (1987) and using several S-values in (4), and then selecting the best fit. The fitted line followed the data points very well for both watersheds, indicating that the total amount of precipitation, minus the initial abstraction, is a good indicator for predicting the runoff. Moreover, the Brigalow watershed, with a fitted maximum storage S of 8 cm had, according to Boughton (1987), clearly less available storage than the Back Creek watershed (S = 40 cm).

United States

Unlike in Australia, where runoff events were selected following a dry period, the runoff events for the northeastern United States occurred in a period between April 1 and November 30, when the moisture contents in the watersheds ranged from very dry to very wet. Initial abstractions for the most shallow soil were, therefore, required and calculated with the Thornthwaite-Mather procedure (Steenhuis and van der Molen 1986). Values needed for this procedure included daily precipitation and potential evapotranspiration, and total available moisture. Since no potential evaporation values were available, the sinusoidal approximation was used, with the highest values in July (Table 2) and lowest (zero evaporation) in January. Total available moisture was calculated from published soil data for the most shallow soil using this method: saturated moisture content minus the wilting point times the depth of the soil above the impermeable layer (Table 2).

By plotting the precipitation minus the initial abstraction calculated with the Thornthwaite-Mather procedure for each storm versus the runoff volume for both Catskill watersheds, a clear trend was visible. However, for the Pennsylvania watershed a number of data points did not follow the general trend. These storms all had a rainfall intensity in excess of 4 cm/h during a 30-min interval and probably produced runoff of the Hortonian type. Because we are interested in the saturated contributing areas, these storms were not included in the analysis.

For the Pennsylvania watershed, a very good fit was obtained for the runoff data and the SCS equation [(4)] using a value for S of 24 cm (Fig. 3), except for a number of small runoff events that occurred before the most shallow soil was saturated. These runoff events were probably caused by rain falling near and on the river channel. There was much more scatter in the data for the two Catskill watersheds than for the one in Pennsylvania (Figs. 4 and 5). The fitted lines were obtained with the SCS runoff equation [(4)] and using S = 16 cm for the Biscuit Brook watershed and S = 30 cm for the High Falls Brook watershed. The scatter in the data is not surprising. Some of the variation can be attributed to the accuracy of the precipitation data. While in Pennsylvania the rain gauge was on the site itself, the rain gauges in the Catskill watersheds were located 5–12 km away. Also, some runoff might have been caused by high-intensity rainfall events. Because of the spotty nature of these storms, the intensity data from the rain gauges could not be used to discriminate between the storms.

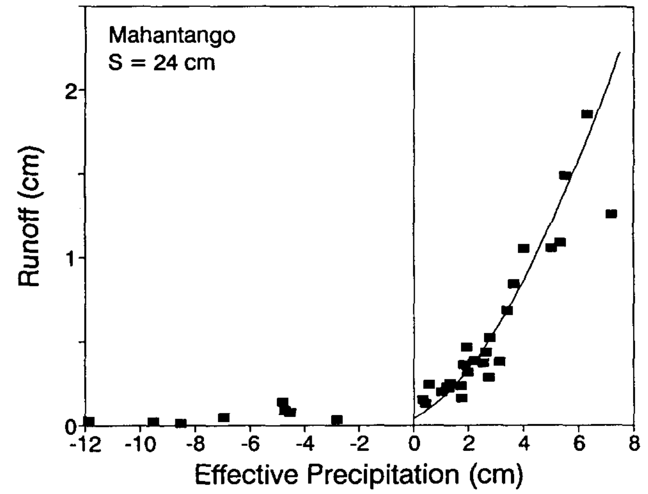

FIG. 3. Observed Runoff for Mahantango Watershed (Squares) and Runoff Predicted with SCS Runoff Equation (S = 24 cm)

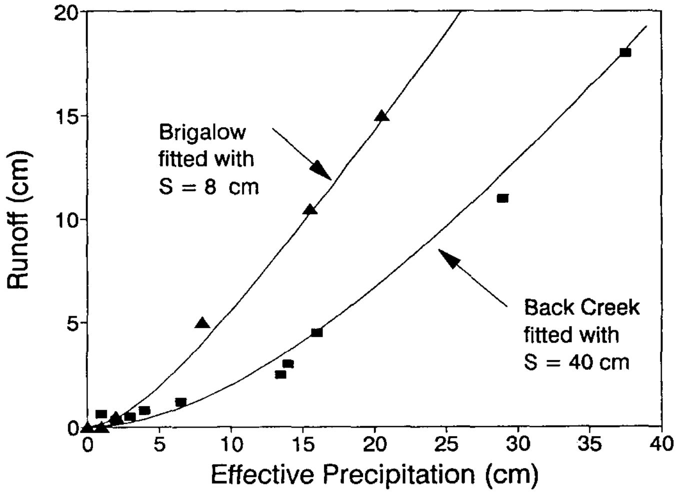

FIG. 2. Observed Runoff for Brigalow (Triangles) and Back Creek (Squares) Watersheds (Boughton 1987)

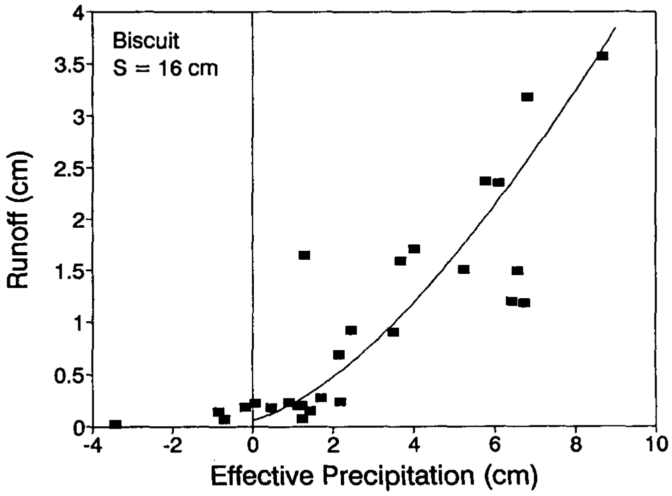

FIG. 4. Observed Runoff for Biscuit Brook Watershed (Squares) and Runoff Predicted with SCS Runoff Equation (S = 16 cm)

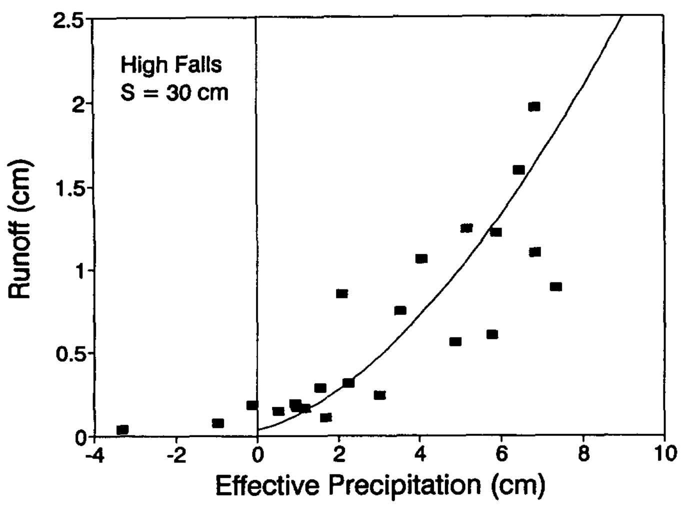

FIG. 5. Observed Runoff for High Falls Brook Watershed (Squares) and Runoff Predicted with SCS Runoff Equation (S = 30 cm)

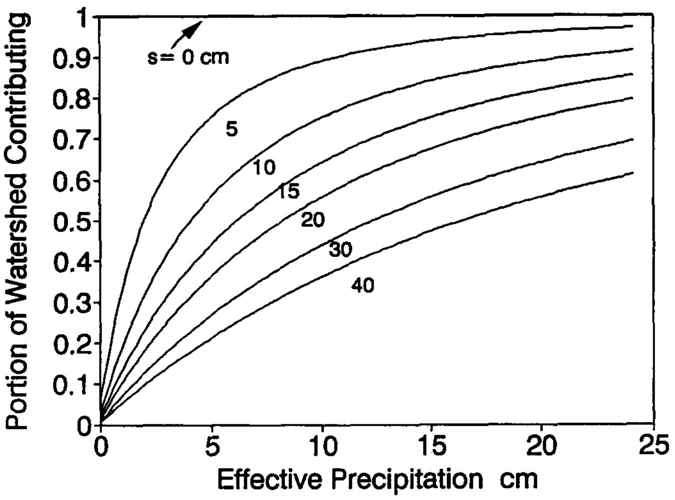

FIG. 7. Plot of Contributing Area versus Effective Precipitation

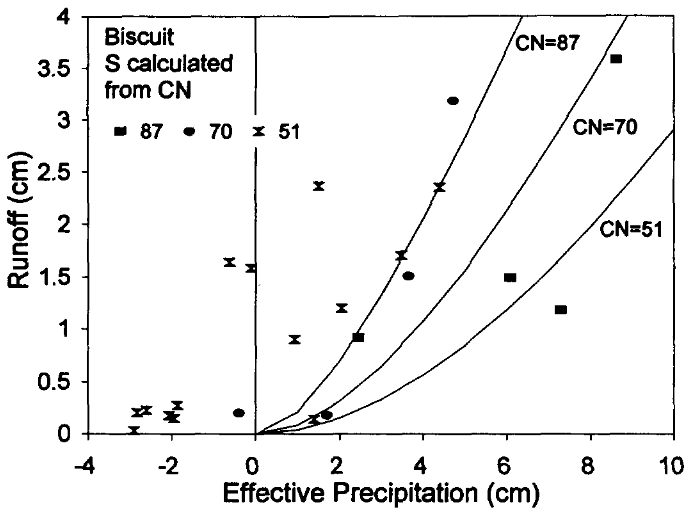

FIG. 6. Observed Runoff for Biscuit Brook Watershed (Solid Lines Represent Runoff Calculated with Curve Number Appropriate for Three Antecedent Moisture Conditions)

TABLE 3. Curve Numbers and S Based on Moisture Conditions (5 Day)

Moisture condition (1)	Curve number (2)	Antecedent Moisture Dormant (cm) (3)	Antecedent Moisture Growing (cm) (4)	S (cm) (5)
I	51	<1.3	<3.6	24.4
II	70	1.3-2.8	3.6-5.3	10.9
III	87	>2.8	>5.3	3.8

COMMENTARY AND CONCLUSIONS

Despite its wide use, the SCS equation is heavily discounted by many hydrologists due to its lack of sound theoretical background. In the process of finding a simple method for determining contributing areas within a watershed, we have established a conceptual basis for the SCS runoff equation in its most elementary form as originally proposed by Mockus [in Rallison (1980)]. By simply assuming that the runoff occurs from areas that are saturated and that the remaining watershed does not contribute any runoff, the SCS equation predicts the contributing area correctly at both limits (P_e = 0 and $P_e = \infty$). In addition, it predicts runoff events well for watersheds (using a constant S-value) that are known to have saturated areas contributing to streamflow.

To better understand if the method we propose for finding the initial abstraction (i.e., the Thornthwaite-Mather procedure for the most shallow soil) is an improvement over the original SCS equation where the initial abstraction is calculated as $0.2S$, as determined from the curve number, we plotted, for the Biscuit Brook watershed, the effective rainfall ($P - 0.2S$) against the observed runoff (Fig. 6). The watershed is heavily forested, with soils mainly of hydrologic soil group C. Curve numbers were 87, 70, and 51, for antecedent moisture conditions I, II, and III (see Table 3). Initial abstractions ($0.2S$) were then calculated based on time of year (dormant or growing) and amount of rainfall during the 5 preceding days. The effective precipitation was then calculated, as the amount of rainfall minus the initial abstraction. The effective precipitation was then plotted versus the observed runoff in Fig. 6 (the symbols indicate the antecedent moisture conditions at the start of the event). Curves calculated from the SCS equation are also plotted. The observed runoff for each antecedent moisture condition did not follow as well the trend indicated by the appropriate curve number as it did when we used the Thornthwaite-Mather procedure for calculating the initial abstraction and one curve number for calculating the runoff. Similarly, Hawkins (1979), in Utah, who also tried to relate the SCS curve number to partial saturated areas, found that using $0.2S$ for initial abstraction did not result in good runoff prediction unless S was dependent on rainfall amounts.

Because of the interest in contributing area in water-quality studies, we made a graph relating contributing area as a function of effective precipitation for several S-values (Fig. 7). This information can be used in simulation studies to calculate the probabilities of runoff from saturated areas. These probabilities could be useful for estimates of erosion hazard or of the probability of stream pollution from land-applied manure or recently sprayed pesticides via the overland-flow pathway.

The next step of this approach is to measure saturated areas as a function of precipitation amounts for several low-intensity storms and to compare the predicted contributing areas with the observed ones. This is needed for a more definitive validation, and might lead to some adaptations of the proposed conceptual model for explaining the SCS curve-number approach. However, despite these reservations, it is interesting that the SCS equation in its most elementary form can be derived from partial-area hydrology.

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