An Equation for Describing Solute Transport in Field Soils with Preferential Flow Paths


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Abstract

Modeling solutes under field conditions is cumbersome due to presence of preferential flow paths and the input data needed to describe these paths. We propose a simple equation that can predict the breakthrough of solutes without excessive data requirements. Conceptually the soil is divided in a distribution layer and a conveyance zone. The distribution zone act as a linear reservoir resulting in an exponential loss of solutes from this zone. In the conveyance zone the transport of solutes is described with the convective dispersive equation. Input data required are apparent water content of the distribution zone and solute velocity and dispersion in the conveyance zone. The model with these three parameters was able to describe the breakthrough of solutes through undisturbed columns.

Keywords: Solute transport, Miscible displacement, Tracer, Advection, Dispersion, Distribution layer

Introduction

Lawes et al. (1882) first observed that water collected in drains can be separated into two constituents: preferential flow (direct drainage) and matrix flow (general drainage). Preferential flow is the rapid and local transport of water and solutes in soils (Stagnitti et al., 1994). Matrix flow is the relative slow and even movement of solutes through the soil. Kung noted that under steady state conditions the solute transport behavior was surprisingly similar for four different sites in the US (Kung et al., 2000). This means that there must be one equation for describing the solute transport in field soils in which water and solutes can move through preferential transport paths. However, no generally accepted analytical expression for describing this type of solute transport has been agreed upon. In this paper, we propose and test such an expression.

Recently, it has been noted that for describing solute movement, the soil profile can be distinguished in both a distribution zone near the soil surface and a conveyance zone below (Steenhuis et al., 1994; Ritsema et al., 1995) The distribution zone funnels water and solutes in flow paths of the conveyance zone (Steenhuis et al., 1994). The thickness of the distribution zone depends on land use and tillage practices (Fig. 1). For brevity, we will assume that the conveyance has only one set of flow paths through which the water flows with approximately the same velocity. However, without loss of generality, the theory can be extended, equally well, for two (matrix and preferential) or more flow regimes.

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Mathematical Description

The soil profile consists of a distribution and preferential zone: The solute concentration in and leaving the distribution zone is equal and can be described with an initial $C_0$ as (Steenhuis et al., 1994):

$$ C = C_0 \exp(-\lambda t) \quad (1) $$

where $\lambda$ is the coefficient equal to $q/w$; $q$ is the steady state flow rate; $w$ is the apparent water content of the distribution zone and equals $d(\rho k_d + \theta_s) / d$; $d$ is depth of distribution zone; $\theta_s$ is the saturated moisture content; $\rho$ is bulk density; and $k_d$ is the adsorption partition coefficient. In case water is added with a solute concentration $C_0$, the concentration in the water leaving the distribution zone is:

$$ C = C_0 \left( 1 - \exp(-\lambda t) \right) \quad (2) $$

Equations (1) and (2) are similar to that used by the U.S. Environmental Protection Agency (USEPA, 1992) in predicting the loss of metals from the incorporation zone of sludge.

It is reasonable to assume that the transport in the preferential flow paths of the conveyance zone can be described with the convective-dispersive equation viz (Parker and van Genuchten, 1984):

$$ \frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial x^2} - V \frac{\partial C}{\partial t} \quad (3) $$

where $V$ is the velocity of the solute and equals $q/(\beta(\rho k_d + \theta_s)$, where $\beta$ is the mobile region fraction and $D$ is the dispersion coefficient. Using Laplace transforms, Eq. (3) can be integrated subject to the boundary condition described in Eq. (1) and initially no solutes in the column for $4D\lambda/V^2 < 1$ as (see also Toride et al., 1995):

$$ C = \frac{1}{2} C_0 \exp(-\lambda t) \left[ \exp \left( \frac{Vx}{2D} (1 - \alpha) \right) \text{erfc} \left( \frac{x - Vt\alpha}{\sqrt{4Dt}} \right) + \exp \left( \frac{Vx}{2D} (1 + \alpha) \right) \text{erfc} \left( \frac{x + Vt\alpha}{\sqrt{4Dt}} \right) \right] \quad (4) $$

where $\alpha = \sqrt{1 - \frac{4D\lambda}{V^2}}$

This solution reduces, as expected, to the convective-dispersive equation when $\lambda = 0$ and $C = C_p$. Similar to the convective-dispersive equation, the last two terms can usually be neglected when $x$ or $t$ are sufficiently large, i.e., $(x + V t \alpha)/(4 D t)^{3/2} > 3$. For boundary condition 2, we find by superposition for sufficient large $x$ and $t$, that the concentration in the conveyance zone equals:

$$ \frac{x - Vt}{\sqrt{4Dt}} - \exp \left( \frac{Vx}{2D} (1 - \alpha) - \lambda t \right) \quad (5) $$

Equation (5) is valid for continuous application of the solute. For a pulsed application where the solute in the rainwater is switched back to water at time $t = \tau$, the solute concentration for $t > \tau$ can be found by subtracting the concentration calculated with Eq. (5) at $t - \tau$ from the concentration calculated at time $t$.

Application

The experiments are used as examples to test the validity of the equations. The first example is example (4b) in Parker and van Genuchten (1984) that was employed in testing the

Figure 2. Observed and predicted boron concentration with two-region model (Parker van Genuchten, 1984) and “predicted” with superposition of Eq. (5).
two-region model. This experiment considered the movement of boron through a Glendale clay loam. Boron was added at a concentration of 20 mg/l for 5.06 days to an initially solute free soil column of 30 cm. The measured pore water velocity was 38.5 cm/day (Parker and van Genuchten, 1984). The applied flux is not given but can be estimated as approximately 12 cm/d. The observed results are shown in Fig. 2 together with the predicted results of the two-sided model (thin line) and with Eq. (5) for a pulsed application (thick line). The parameters used for fitting Eq. (5) were \( w = 22 \) cm, \( V = 30 \) cm/day, and dispersivity \( (D/V) \) of 1. Equation (5) fits the data (with three fitting parameters) just as well as the two-region model (with four parameters to fit, Parker and van Genuchten (1984)). Consequently, the comparison only shows that Eq. (5) is one of the models that can fit the data. Of interest, therefore, is to examine if the model results are physically valid. The models predict that there is no or very little adsorption in the conveyance zone (velocity of the chloride nearly equal to that measured). This is in accordance of the experiments at Beltsville, in which solutes with different adsorption partition coefficients were found at the same depth (Kung et al., 2000). Moreover, the results of a sensitivity analysis showed that the apparent water content is the most sensitive parameter and is in agreement with experimental findings that the thickness of the distribution zone greatly affects the outflow pattern (Steenhuis et al., 1994). Finally, an apparent water content of 22 cm indicates that adsorption is important. The precise estimate of the adsorption partition coefficient will require an independent measurement of the depth of the distribution zone.

A second experiment to test the model’s validity is given by Akhtar et al. (2000) in which a 5.5 cm application of 13.5 meq/l LiCl solution was applied to 39 -30 cm wide and 40 cm long undisturbed columns of an Arkport sandy loam. The salt was leached out with two daily 5.5 cm water additions followed with weekly 5.5 cm irrigations. An oat crop was seeded 3 weeks after the initiation of the experiment. The observed breakthrough curve of the 39 columns, together with the predicted outflow concentration are shown in Fig. 3 as a function of the water applied. The most sensitive parameter is, again, the apparent water content \( (w = 8 \) cm). Velocities (in this case mobile water fractions because reduced variables were used) and dispersivities in the subsoil have little effect as long as it is above a critical value (alpha remaining close to 1) on the breakthrough characteristics. The thin line is a mobile fraction of 5%; the thick line 2% and the 1% line coincided with the 2% line. Dispersivities in all cases were equal to 1.

The equation was also tested for an experiment by Gish, Kung and coworkers in which solute leaching to tile lines was studied with two different flow rates producing greatly different breakthrough curves. Equation (5) was able to reproduce both hydrograph by only changing the velocity of the water in the conveyance zone. This is the topic of a future paper. It seems, therefore, that the model proposed here has a great potential to predict solute leaching under field conditions in soils at or above field capacity. Modification for dry soils are being studied.

References


