Comment on "Series solution for Richards equation under concentration boundary conditions and uniform initial conditions" by Guido Daniel Salvucci

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Recently, *Salvucci* [1996] produced an interesting series solution for fixed surface moisture content θ_1 and uniform initial water content θ_0 and, in particular, produced a convenient infiltration equation, equation (38) in that paper. *Barry et al*. [1995] also produced an infiltration equation, which can be applied to the particular conditions used by *Salvucci* [1996], although it was not discussed by him. Two very similar equations were recommended by *Barry et al*. [1995], equations (14) and (17)]. For instance, their (17) reads

$$I^* = t^* + 1 - \gamma + \frac{\gamma}{1+t^*}\left\{\left[\exp - \frac{2t^*}{3}\right] \cdot [1-(1-\gamma)^8 t^{*5/2}] + (2\gamma + t^*)\ln\left(1+\frac{t^*}{\gamma}\right)\right\} - \exp\left[-\frac{\sqrt{2t^*}}{1+\sqrt{2t^*}/6} - \frac{2t^*}{3}\right] \quad (1)$$

with, for Salvucci's conditions,

$$I^* = (I - k_0 t)2(k_1 - k_0)/S^2 \quad (2)$$

$$t^* = 2t(k_1 - k_0)^2/S^2, \quad (3)$$

and γ is a soil characteristic varying between 0 and 1, 1 being the value for a Green and Ampt soil and 0 for a Gardner soil [*Barry et al*., 1995]. I is the cumulative infiltration, t is the time, k_1 and k_0 are the values of conductivity for $\theta = \theta_1$ and $\theta = \theta_0$, respectively, and S is the sorptivity. A slightly simpler equation [*Barry et al*., 1995, equation (14)] can also be used with no significant numerical difference.

With the same notations, Salvucci's (38) reads

$$I^* = \sqrt{2t^*/(1+t^*)} + \tfrac{1}{2}t^*/[1 + t^* - \sqrt{t^*(1+t^*)}]. \quad (4)$$

Salvucci [1996, Figure 4] applied his result to Yolo light clay as done earlier by *Barry et al*. [1995, Figure 7]. It is clear that (1) is far more accurate than (4). Figure 1 here compares (1) and (4) with $\gamma = 0.341$ (from the property values of Yolo light clay given by *Barry et al*. [1995]).

Figure 1 here and Figure 4 of Salvucci [1996] show that (1) is in essential (within 1%) agreement with the numerical solution of *Salvucci* [1996] (Figure 7 of *Barry et al*. [1995] already showed its agreement with the numerical solution of that paper). For Yolo light clay, (4) has a maximum error of around $7\frac{1}{2}$% as also shown by *Salvucci* [1996, Figure 4].

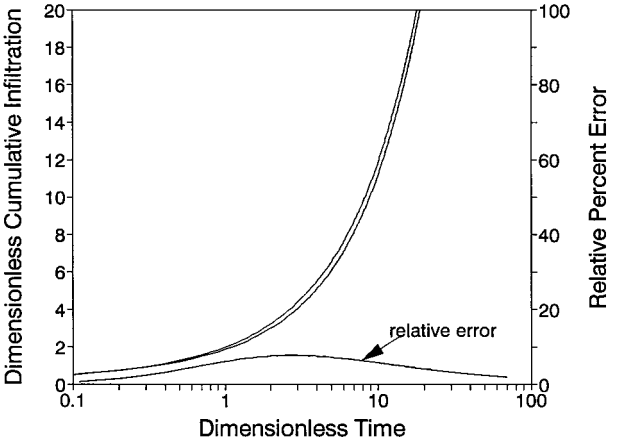

Figure 1. Dimensionless cumulative infiltration I^* as a function of the dimensionless time t^*. The top curve is from (1), and the bottom curve is from (4). The relative error of (4) is also given.

It would be interesting to find out whether the inclusion of more terms in Salvucci's series would reduce the error to <1%, as with the solution of *Barry et al*. [1995].

References

Barry, D. A., J.-Y. Parlange, R. Haverkamp and P. J. Ross, Infiltration under ponded conditions, 4, An explicit predictive infiltration formula, *Soil Sci*., *160*, 8–17, 1995.

Salvucci, G. D., Series solution for Richards equation under concentration boundary conditions and uniform initial conditions, *Water Resour. Res*., *32*, 2401–2407, 1996.

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Paper number 98WR01834.
0043-1397/98/98WR-01834$09.00

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(Received November 20, 1996; revised January 14, 1998; accepted May 27, 1998.)