Cutting Across Disciplines

Vadose Zone Hydrology
Soil Properties and Water Movement

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For all spatial scales, from pore through local and field, to a watershed, interaction of the land surface with the atmosphere will be one of the crucial topics in hydrology and environmental sciences over the forthcoming years. The recent lack of water in many parts of the world shows that there is an urgent need to assess our knowledge on the soil moisture dynamics.

The difficulty of parameterization of soil hydrological processes lies not only in the nonlinearity of the unsaturated flow equation but also in the mismatch between the scales of measurements and the scale of model predictions. Most standard measurements of soil physical parameters provide information only at the local scale and highlight the underlying variability in soil hydrological characteristics. The efficiency of soil characteristic parameterization for the field scale depends on the clear definition of the functional relationships and parameters to be measured, and on the development of possible methods for the determination of soil characteristics with a realistic use time and effort.

The soil’s hydraulic properties that affect the flow behavior can be expressed by a soil water retention curve that describes the relation between volumetric water content, \( \theta(L^3L^{-3}) \), and soil water pressure, \( h(L) \), plus the relation between volumetric water content and hydraulic conductivity, \( K(L/T) \).

In the next section, the determination of soil hydraulic parameters is first discussed for local and field scale. Then, we show how the pore-scale processes can be linked to soil hydraulic properties. These properties are then used in some of the modern methods that use integral and superposition solutions of Richards’ equation for infiltration and water flow problems for both stable and preferential types of
Determining Soil Hydraulic Properties

The methods for the movements of soil hydraulic properties can be classified in two fundamentally different categories: (1) those which make point measurements of the soil hydraulic characteristics at a given number of locations over the field and then scale these "local" measurements upward to field-averaged soil hydraulic parameters; and (2) those which calculate the field-averaged characteristic parameters by the use of inverse methodology from flow phenomena representative of the field scale.

Local Scale

Two types of approaches can also be distinguished for the local determination of the unknown soil parameters: direct or indirect. In general, direct experimental measurement techniques rely on precise and time-consuming experimental procedures, such as the instantaneous profile method (Rose et al., 1969), and these are usually not suitable for routine application at a great number of measurement points over the field. As a result, other field techniques have been developed.

The most common is perhaps the disc infiltrometer method, which uses observations of three-dimensional infiltration rates at given initial and boundary conditions. In most cases, slightly negative supply pressures are applied (e.g., Clothier and White, 1981). The fact that the initial and boundary conditions are well controlled makes these experiments particularly appropriate for data analysis through inverse procedures. The standard analysis uses Wooding's (1968) solution for the three-dimensional steady-state infiltration, which is valid for infinite time and uniform initial conditions. Unfortunately, neither of these conditions are often met in the field (Haverkamp et al., 1994; Smettem et al., 1995). In order to overcome these limitations, a three-dimensional analytical solution of infiltration has been recently derived (Haverkamp et al., 1994; Smettem et al., 1994). This allows a description of transient three-dimensional infiltration behavior. The analysis of disc infiltrometer observations with this transient solution allows better point estimation of the hydraulic conductivity and sorptivity for a set of chosen initial and boundary conditions.

This analytical solution is valid over the entire time range, and is based on the use of parameters with sound physical meaning. It is adjustable for varying initial and boundary conditions. This point is crucial for it permits deconvolution of the integral coefficients, such as sorptivity, to generic soils hydraulic parameters (Connell et al., 1998).

Many attempts have been made in the literature to predict the soil characteristics from more easily measurable soil data. The most appealing alternative approach consists in predicting the soil properties from textural and structural soil data. The simplest empirical approach consists of relating water contents at specific soil pressure values to particle size percentages, organic matter content, and bulk density, using statistical regression analyses (e.g., Gupta and Larson, 1979; Rawls and Brakensiek, 1982). Instead of estimating only discrete $h(\theta)$-points, other authors (e.g., Clapp and Hornberger, 1978) have tried to predict the different water-retention fitting parameters directly from textural and structural properties. Validity of these models is limited to the soil tested in each study. When used for prediction, however, they may lead to significant errors in water content, especially in the wet range of $h(\theta)$. Nevertheless, and in spite of these discouraging results, closer analysis clearly showed the existence of a correlation trend between the shape parameters of the water-retention curve and that of the cumulative particle size distribution function. This result formed the basis of two physico-empirical-based models proposed in the literature (Arya and Parule, 1981; Haverkamp and Parlane, 1986). The former study yields discrete point values of $h(\theta)$, while the model proposed in the latter allows the prediction of water-retention curves for the very restrictive group of nonstructured sandy soils by fitting of a van Gencuchen (1980) type equation to the particle size distribution. The results confirmed the validity of the hypothesis that there is a shape similarity between the water-retention curve and the cumulative particle size distribution function.

Field Scale

The second and larger scale approach calculates field-averaged soil hydraulic parameters by the use of inverse techniques from flow phenomena representative of the field scale. This approach relates observations of flow processes at the field scale to soil hydraulic properties. It is frequently used in the field of irrigation engineering (e.g., Katopodes et al., 1990; Bautista and Wallender, 1993). The overland flow processes encountered during border, and/or furrow irrigation, experiments can be precisely monitored, which makes them particularly appropriate for the application of inverse procedures.

Irrigation flow is a combined problem of infiltration-overland flow. Mostly, measurements of the advance rate of the surface surge during the irrigation advance phase are used to calculate the field-averaged infiltration characteristics. Over the last decade, elaborate numerical solutions for the infiltration-irrigation advance problem are being used more and more frequently (e.g., Katopodes and Steflkoff, 1977; Schmitz and Seus, 1990). However, it is questionable whether numerical surface flow models are really needed to describe the irrigation flow process when dealing with the simulation and/or prediction of the irrigation advance phase under actual field conditions. Great uncertainties exist for field-averaged infiltration parameters, and these can totally dominate the flow process during the advance phase, so it is sufficient to consider just volume-balanced hydrological approaches (e.g., Philip and Farrell, 1964; Parlane, 1973).

However, it is most surprising that, over the last three decades, researchers in the field of irrigation engineering have persisted to treat the subsurface flow aspects in a rudimentary way. The infiltration has been invariably calculated by purely empirical quantitative infiltration equations (e.g., Kostiakov (1932) equation). These lack any physical meaning. As these infiltration equations are only valid for the conditions...
Thus, the conductivity $K$ is estimated by

$$\frac{K}{K_s} = \int_{0}^{1} \frac{R}{\theta - \theta_0} d\theta,$$

where $K_s$ and $\theta_s$ are the saturated values of $K$ and $\theta$, respectively, and $R$ is required for the initial and boundary conditions. An accurate approximation of the full-time solution of the infiltration equation is achieved by using a physically based infiltration model (Haverkamp et al., 1995) and a fully distributed approach. A first-order approximation of the boundary conditions allows the use of physically based infiltration models. The second-order solution is obtained by applying the infiltration equations in each increment.

In conclusion, it can be stated that the use of physically based infiltration models plays a key role in determining the infiltration characteristics of the soil. The precision of the infiltration equations, which includes the use of physically based infiltration models, is crucial in order to guarantee successful use of inverse procedures. To determine the hydraulic parameters, independent of initial and boundary conditions, that have sound physical meaning and which can be adjusted for varying initial and boundary conditions.

Next, we will develop some analytical forms that describe the dependence of soil hydraulic properties on water content and hydraulic head. The dependency on local and field-scale processes is used to determine the hydraulic parameters that are used in the infiltration equations.
$D = 3$. Finally, it should be mentioned that Perrier et al. (1995) have obtained the
direct calculation of hydraulic properties for computer construction of fractal soils.
So far, we have shown that fractal theory may be used to find the relationship
between unsaturated conductivity and moisture content. Next, based on pore-scale
processes, we examine the dependence of matric potential and moisture content.
Unlike the saturated hydraulic conductivity relation, the matric potential/moisture
content function is hysteretic.

### Hysteresis in the Soil Moisture Characteristic Relationships

Water movement in unsaturated soils is always subject to hysteresis, although its
effects are often masked by heterogeneities. Theoretically, for a given fractal geometry,
it should be possible to predict hysteretic effects from first principles. However,
this problem remains largely unsolved and our rather sketchy understanding of soil
structure suggests that only a few soil models will yield to this approach (a good
summary can be found in Perrier et al., 1995). Instead, hysteresis in soils remains
largely based on Poulouvasilius’s (1962) analysis obtained by applying the independent
domain theory to soils. Hysteresis affects the water-retention curves linking the
matric potential, $h$, to water content, $\theta$. The dependence of $K$ on $\theta$ is unaffected
by hysteresis, for it invariably obeys equation (4.7).

The following analysis here is a greatly simplified theory which requires only one
boundary to predict the other boundary and all scanning curves in between.
Basically, the model of Parlane (1976, 1980) (see also Hogarth et al., 1988 and
Liu et al., 1995) leads to the simple relation between any wetting curve of order
2$n$, $\Theta_w(h, h_{2n})$, and a drying curve of order $2n + 1$, $\Theta_d(h, h_{2n+1})$, which is issued from
it at $h = h_{2n+1}$ and $\theta = \theta_{2n+1}$. In this notation, $\Theta_w(h, h_{2n})$ means that it is issued from
a drying curve of order $(n - 1)$ at $h = h_{2n}$, and $\theta = \theta_{2n}$. Note that we define the wetting
domain for $h_0 \rightarrow -\infty$. Figure 4.1 shows the wetting boundary $\Theta_w(h, h_0)$, plus a first-order drying curve $\Theta_d(h, h_1)$ issued from it at $(\theta_1, h_1)$, as well as a second-order drying curve $\Theta_w(h, h_2)$ issued from it at $(\theta_2, h_2)$, and finally a third-order drying curve $\Theta_d(h, h_3)$ issued from $\Theta_w(h, h_2)$ at $(\theta_3, h_3)$. The relation between wetting and drying curves is governed by

$$\Theta_d(h, h_{2n+1}) = \Theta_w(h, h_{2n}) - [h - h_{2n+1}] \frac{d\Theta_w(h, h_{2n})}{dh}$$

(4.8)

Following the Brooks and Corey (1964) model, the first drying curve, when $\theta_1 = \theta_n$,
will not desaturate until an air entry value $h_{ae}$ is reached. Then, equation (4.8)
indicates that for $h > h_{ae}$, $\Theta_w$ is linear in $h$ (see figure 4.1). For $h < h_{ae}$, which is
again consistent with fractal theory, we take for wetting boundary ($h_0 \rightarrow -\infty$),

$$\frac{\Theta_w(h, h_0)}{\Theta_{Mae}} = \left(\frac{h_{ae}}{h}\right)^\lambda, \quad h \leq h_{ae}$$

(4.9)

Figure 4.1 Sketch of wetting boundary and drying curves of different orders.

where $\Theta_{Mae}$ is simply the value of $\Theta_w$ at $h = h_{ae}$. The general solution to equation
(4.8), is, for $h \leq h_{ae}$,

$$\frac{\Theta_w(h, h_{2n}) - \Theta_{2n}}{\Theta_{Mae}} = \left(\frac{h_{ae}}{h}\right)^\lambda \left[1 + \lambda \frac{h_{ae}}{h_{2n}}\right] - \left(\frac{h_{ae}}{h_{2n}}\right)^\lambda (1 + \lambda)$$

(4.10)

and

$$\frac{\Theta_d(h, h_{2n+1}) - \Theta_{2n}}{\Theta_{Mae}} = \left(\frac{h_{ae}}{h}\right)^\lambda \left[1 + \lambda \frac{h_{ae}}{h_{2n+1}}\right] - \left(\frac{h_{ae}}{h_{2n}}\right)^\lambda (1 + \lambda \frac{h_{ae}}{h_{2n+1}})$$

(4.11)

with straight curves for $h \geq h_{ae}$ (as shown in figure 4.1). Note also in figure 4.1 that
two successive wetting and drying curves share the initial and final points—that is, at
$h = h_{2n}$ and $h = h_{2n+1}$. Parameter $\Theta_{Mae}$ is related to the total porosity, $\epsilon$, by (Liu et al.,
1995) as

$$\frac{\theta_2}{\epsilon} = \left(\frac{h_{ae}}{h_2}\right)^\lambda \left[1 - \frac{h_1}{h_2} + \frac{h_1 \lambda}{h_2(1 + \lambda)}\right]$$

(4.12)

Hence, if we know the starting point of any curve, for example, $(\theta_n, h_n)$ in equation
(4.10), and the porosity $\epsilon$, then $\lambda$ and $h_{ae}$ are given by the condition that
$\Theta_d(h_2, h_1) = \theta_2$, or, from equation (4.11),

$$\frac{\theta_1}{\epsilon} = \left(\frac{h_{ae}}{h_2}\right)^\lambda \left[1 - \frac{h_1}{h_2} + \frac{h_1 \lambda}{h_2(1 + \lambda)}\right]$$

(4.13)

and the slope of the line formed by the starting points of the first and third drying
curves is equal to $d\Theta_w(h, h_2)/dh$ at $h = h_{ae}$, or, from equation (4.11),

$$\frac{\theta_3 - \theta_1}{\epsilon} = \frac{e \lambda}{h_2 - h_1} \left[1 - \left(\frac{h_{ae}}{h_2}\right)^\lambda - 1\right]$$

(4.14)
SOIL WATER DIFFUSIVITY AS

The gravity effect and repetitive of $\phi$ by $z$ in equation (4.10). In equation (4.11) we can define the


diffusivity associated with the rapid changes of $G$ and can be differenced.$

So the appropriate surface boundary condition is $y = 0$ or

$\frac{\partial y}{\partial z} = 0$

where $y$ is the top potential.

Next, we will discuss how these properties can be incorporated in several scales. Next, we will discuss how these properties can be incorporated in several scales.

In this section, we have examined ways of obtaining similar hydraulic properties at

equal depth.

It is evident that, within the scale of the dam, the flow of two choices of the parameters

is different. However, in this case, $\phi = 1$ and $\phi = 6$ are also shown on the figure.

Solutions to the equations in Figure 4.11 are very well with the boundary conditions

of equation (4.10) and (4.11). The predicted boundary conditions of (4.1) and (4.12) are

as follows:

\[ \phi = \frac{1}{2} \quad \text{and} \quad \phi = 6 \]

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Finally, the effect of nacropoles on water and source flow is addressed.

Then, this section of material on the flow of two choices

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\[ \phi = \frac{1}{2} \quad \text{and} \quad \phi = 6 \]

Water Movement—Local and Field Scale

Figure 4.2 Measured Wetting Boundary and Drying Curves Compared with Predicted

Moisture Potential (cm)
Bruce and Klute (1956) obtained a fundamental similarity equation, taking normalized moisture content, \( \vartheta \), at \( t = 0 \) constant, which can be taken as zero, with \( \vartheta \) and \( x = 0 \) constant at all times, which for simplicity we normalize to 1, at no loss of generality. With the only spatial variable being \( x \), equation (4.16) reduces to the well-known Bruce and Klute equation,

\[
D = \frac{1}{2} \frac{d \Phi}{d \theta} \int_{0}^{1} e^{\theta} \ d \theta
\]

where \( \Phi = x r^{-1/2} \) is the Boltzmann similarity variable.

Probably the best understanding of the structure of any solution of equation (4.19) is based on the analytical expansion technique of Heaslet and Alksne (1961) which was applied by Parlane et al. (1992a). This technique balances accuracy with simplicity. Keeping two terms in the expansion yields

\[
2 \int_{0}^{1} \frac{D}{\theta} \ d \theta = S \Phi + \frac{A}{2} \vartheta^2
\]

where \( S \) is the sorptivity and \( A \) is a small number for standard soils. The integrand on the left-hand side is \( D/\theta \), which can come naturally out of iterative schemes, under very general conditions, as, for example, that of equation (10) in Parlane (1972). All modern analytical approximations to solutions of Richards' equation use integrated forms of the latter. The Bruce and Klute equation is the result of one integration, but a second integration yields

\[
\Phi = 2 \int_{0}^{1} \frac{D}{\Phi} \ d \theta \int_{0}^{1} e^{\theta} \ d \theta
\]

of which equation (4.20) is clearly an approximation. Hence, all these new solutions are sometimes said to result from a double integration technique. One particular double integral, obtained from equation (4.19), is that of Parlane (1975a):

\[
2 \int_{0}^{1} D \ d \theta = \int_{0}^{1} \Phi \ d \theta
\]

so that with equation (4.20),

\[
S^2 = 2 \int_{0}^{1} D \ d \theta \left( 1 - \frac{A}{2} \right)
\]

Thus, the solution requires one more equation to calculate both \( S \) and \( A \). Many accurate relations have been proposed in the past relating the sorptivity and \( D \) (Parlane et al., 1994). An early expression (Parlane, 1975b) gives nonetheless very simple, yet precise results (Elrick and Robin, 1981), namely

\[
S^2 = \int_{0}^{1} (1 + \theta) D \ d \theta
\]

Table 4.1 gives values of \( D_{0}^{1/2} \) for a power law diffusivity, \( D = D_{0} \theta^A \), which shows a significant improvement of equation (4.25) over equation (4.24), albeit at the cost of calculating \( \int \theta^A D \ d \theta \).

The same technique of double integration can be used even when gravity is present. Assuming again that movement takes place in the z-direction only, we first consider a sudden change in the normalized water content at the surface, from 0 to 1, and we approximate \( D \) by a Dirac delta-function. So, there is a wetting front where the water content jumps from zero to one, and \( A = 0 \) in equation (4.23). In one dimension we can write, for equation (4.16),

\[
\frac{D \ d \theta}{D \ d \theta} = \frac{D \ d \theta}{D \ d \theta} \frac{d \vartheta}{d \vartheta}
\]

or, by two successive integrations,

\[
q_1 = \int_{0}^{1} D \ d \theta - \int_{0}^{1} K \ d \theta
\]

where \( \int D \ d \theta \) can be replaced by \( S^2/2 \), since \( D \) is a delta-function. Here, \( I \) is the cumulative infiltration and \( q \) is the flux at \( z = 0 \). To integrate the last term, the behavior of \( K \) needs to be known as \( \theta \to 1 \). For instance, if \( dK/d\theta \) is finite as \( \theta \to 1 \), then equation (4.28) becomes

\[
q_1 = S^2/2 + K_1 I
\]

Table 4.1 Exact and Approximate Values of the Sorptivity, \( S_{0}^{1/2} \), when \( D = D_{0} \theta^A \) for Various \( \lambda \) Values

<table>
<thead>
<tr>
<th>( \lambda )</th>
<th>Exact</th>
<th>Equation (4.25)</th>
<th>Equation (4.24)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.128379</td>
<td>1.128378</td>
<td>1.2247</td>
</tr>
<tr>
<td>1</td>
<td>0.887496</td>
<td>0.887618</td>
<td>0.9128</td>
</tr>
<tr>
<td>2</td>
<td>0.753052</td>
<td>0.753097</td>
<td>0.7637</td>
</tr>
<tr>
<td>3</td>
<td>0.665166</td>
<td>0.665187</td>
<td>0.6708</td>
</tr>
<tr>
<td>4</td>
<td>0.600130</td>
<td>0.602138</td>
<td>0.6055</td>
</tr>
<tr>
<td>5</td>
<td>0.554122</td>
<td>0.554123</td>
<td>0.5563</td>
</tr>
<tr>
<td>6</td>
<td>0.519996</td>
<td>0.515997</td>
<td>0.5175</td>
</tr>
<tr>
<td>7</td>
<td>0.484778</td>
<td>0.484779</td>
<td>0.4859</td>
</tr>
<tr>
<td>8</td>
<td>0.458612</td>
<td>0.458612</td>
<td>0.4594</td>
</tr>
<tr>
<td>9</td>
<td>0.436264</td>
<td>0.436265</td>
<td>0.4369</td>
</tr>
<tr>
<td>10</td>
<td>0.416892</td>
<td>0.416893</td>
<td>0.4174</td>
</tr>
</tbody>
</table>
or, by integration,
\[
K_{1t} = I - \frac{S^2}{2K_1} \ln[1 + 2K_1/S^2]
\]
(4.30)

This is the well-known Green and Ampt (1911) result, where \( K_1 = K(\theta = 1) \). A slightly different way to obtain the same result is to start with the flux \( q(\theta, t) \) at any point \( z \),
\[
q(\theta, t) = -D \frac{\partial \phi}{\partial z} + K
\]
(4.31)

or
\[
\frac{\partial z}{\partial \theta} = \frac{D}{K - q(\theta, t)}
\]
(4.32)

If, and only if, \( D \) is a delta-function, \( \partial z/\partial \theta = 0 \), unless \( \theta \to 1 \). So that now, \( q(\theta, t) \) can be replaced by \( q \). Integration of equation (4.32) in that case then gives
\[
I = \int_0^1 D \frac{d\theta}{q - K}
\]
(4.33)

In the Green and Ampt case, where \( D \) is a delta-function, but \( K \) is well behaved, then the integral can be replaced by \( \int D d\theta / (K_1 - q) \), thereby yielding equation (4.29) at once. When \( D \) is a delta-function, it is possible to obtain other results, besides equation (4.29), if \( K \) and \( D \) can be related. An infinite number of relations can be postulated and some have been studied. The best known is that due to Gardner (1958):
\[
K_1 D = \left( \int_0^1 D d\theta \right) dK/d\theta
\]
(4.34)

which gives at once, from equation (4.33), by replacing \( \int D d\theta \) with \( S^2/2 \),
\[
K_{1t} = I + \frac{S^2}{2K_1} \left[ \exp(-2K_1/S^2) - 1 \right]
\]
(4.35)

This expression, first derived by Talasma and Parlane (1972), is an example of a result where \( D \) is a delta-function which is different from the Green and Ampt result of equations (4.29) or (4.30). This result makes it all the more surprising that the Green and Ampt result is sometimes seen as "the" delta-function result, that is to say "the only one" (Barry and Parlane, 1994; Barry et al., 1994, 1996). If one expands both results in equations (4.30) and (4.35) for short times, then
\[
I = S^2/2 + Bt + \ldots
\]
(4.36)

where \( B = 2/3 \) for equation (4.30), and 1/3 for equation (4.35). It is well known that, in most cases, \( B \) is in fact close to 1/3 (Talasma, 1969), which shows the superiority of Gardner's equation (4.34), and the resulting equation (4.35) of Talasma and Parlane (1972). It is also easy to obtain interpolations between equations (4.30) and (4.35) for the intermediate behavior of \( K_1 \), as was done by Parlane et al. (1982). However, an infinite number of other interpolations can be obtained. For instance, we can replace equation (4.34) by the more general

\[
K_{1t} + D = \left( \int_0^1 D d\theta \right) (n+1) K^n dK/d\theta
\]
(4.37)

Or, from equation (4.33), we obtain another "delta-function" result,
\[
I = \frac{S^2 n + 1}{2} \int_0^1 \frac{D}{K_1^{n+1}} dK/d\theta
\]
(4.38)

Clearly, for \( n = 0 \), equation (4.35) follows, and for \( n \to \infty \), \( K_1 \to \) delta-function and equation (4.30) follows. Intermediate behavior will result for any other \( n \) value. In particular, \( B \) in equation (4.36) is given by \( B = 2(n+1)/(n+2)K_1 \) which has the proper limits for both \( n = 0 \) and \( n \to \infty \). For instance, if \( B = 1/2.8 \) (Talasma, 1969), \( n = 0.15 \). Obviously, other functional dependencies besides equation (4.37) can be invented, all giving new "delta-function" results.

Haverkamp et al. (1990) used a different method of interpolation by taking
\[
K = K_1, \quad \text{for} \ h > h_1
\]
(4.39)

This Green and Ampt behavior was achieved by taking another \( K \) for \( h < h_1 \). As earlier, \( h_1 \) here is the water entry value which can be related to the bubbling pressure. Although Haverkamp et al. (1990) took a behavior between that of Gardner and that of Green and Ampt, in the following we limit ourselves to equation (4.34), with \( D \) being a delta-function. Then, equation (4.33) gives at once,
\[
I = \frac{K_1 |h_1| + S^2 - 2K_1 |h_1|}{2K_1} \ln \frac{q}{q - K_1}
\]
(4.40)

Note that in equation (4.40) we replace \( \int D d\theta \) by \( S^2/2 - K_1 |h_1| \), since only values for \( h < h_1 \) enter \( \int D d\theta \). In many respects, the present interpolation is a "natural" one, for \( h_1 \) has a physical reality and for \( h < h_1 \) a Gardner soil is realistic. This \( K \) behavior can be seen as a slight improvement on a Gardner soil. It is only a slight improvement because \( h_1 \) is generally small. This was apparently first considered by Rijtema (1965). For \( h_1 = 0 \), equation (4.35) follows, and for \( S^2 = 2K_1 |h_1| \), equation (4.30) follows. Hence, equation (4.40) can be seen as another "delta-function result," even though Haverkamp et al. (1990) argued that it applied even when \( D \) is not a delta-function for \( h < h_1 \).

So far, we have discussed mainly cumulative infiltration when the water pressure at the soil at the surface is constant, and zero or negative so that the soil remains unsaturated. Next, we will study the infiltration pattern when the water level becomes positive so that part of the soil is saturated and part unsaturated.

**Infiltration—Ponded Surface Condition**

Consider a water layer, \( h_{surf} > 0 \), on the soil surface. Here, \( h_{surf} \) can be an arbitrary function of time. The only effect of this layer is to act as \( |h_1| \) does, so that equation (4.40) becomes

\[
I = \frac{K_1 |h_1| + h_{surf}}{q - K_1} + S^2 - 2K_1 |h_1| \ln \frac{q}{2K_1}
\]
(4.41)
The dependence of \( I \) on \( h_{surf} \) for short times was first examined by Green and Ampt (1911). They obtained, in agreement with equation (4.41),

\[
I^2 = [S^2 + 2K_i h_{surf}]t 
\]  
(4.42)

This result is obviously exact when \( D \) is a delta-function. Small corrections can be introduced when \( D \) is not a delta-function (Parlange et al., 1985, 1992b; Broadbridge, 1990; Barry et al., 1992). However, these corrections are rarely needed in practice.

In general, equation (4.41) requires a numerical solution if \( h_{surf} \) is not constant. But if it is constant, it is possible to obtain \( I(t) \) very accurately (Barry et al., 1995), as

\[
I^* = I^* + 1 - \gamma + \frac{\gamma}{1 + I^*} \left\{ \exp \left( -\frac{2I^*}{3} \right) + (2\gamma + I^*) \ln \left( 1 + \frac{I^*}{\gamma} \right) \right\} - \exp \left[ -\frac{\sqrt{2\gamma}}{1 + \sqrt{\gamma/3}} - \frac{2I^*}{3} \right] 
\]  
(4.43)

where \( I^* \) and \( I^* \) are the dimensionless quantities,

\[
I^* = (I - K_i t)2(K_i - K_s)/[S^2 + 2K_i h_{surf}(\theta_i - \theta_s)] 
\]  
(4.44)

\[
t^* = 2t(K_i - K_s)^2/[S^2 + 2K_i h_{surf}(\theta_i - \theta_s)] 
\]  
(4.45)

with

\[
\gamma = 2K_i h_{surf} + |h_1|(\theta_i - \theta_s)/[S^2 + 2K_i h_{surf}(\theta_i - \theta_s)] 
\]  
(4.46)

Here, \( \theta_i \) and \( \theta_s \) are the “real” initial and saturated water content, and not taken (as before) as being normalized to 0 and 1; \( K_i \) and \( K_s \) are the corresponding conductivities. The advantage of having \( I(t) \) explicitly is that it makes curve fitting of data much easier to determine soil-water parameters, like \( S^2 \), \( K_i \), and \( \gamma \).

Figure 4.3 illustrates the validity of equation (4.43) for infiltration into a coarse river sand (Culligan et al., 1998). There, \( h_{surf} = 3.2 \text{ cm} \), and \( |h_1| = 37.75 \text{ cm} \) was determined from the bubbling pressure of 6.7 cm necessary for air to bubble out when \( h_{surf} = 3.2 \text{ cm} \). Also, \( K_i = 0.015 \text{ cm/s} \), \( \theta_i = 0.373 \), and \( \theta_s = 0.0 \). The only curve-fitted parameter is the value of the sorptivity; we took \( S = 0.47 \text{ cm/s}^{1/2} \). The agreement is excellent at all times. By comparison, and to show the sensitivity of \( I(t) \) to \( h_1 \), the predicted value of \( I \) for \( h_1 = 0 \) is also given. Clearly, even for small \( |h_1| \), namely \( 3.5 \text{ cm} \), its effect is not negligible for long times.

So far, we have discussed the structure of one-dimensional wetting fronts with and without gravity. More complex flows are now discussed which may also require more than one spatial variable.

Superposition Principle

In groundwater studies, superposition is a powerful technique to find new solutions from solutions with simpler boundary conditions. If Richards’ equation were linear, which it is not, it would also be possible to superpose solutions. Then, for instance, we could analyze the interaction of a wetting front with an impervious wall, from the solution in an unbounded soil.

Figure 4.3 Observations of cumulative infiltration in a river sand as a function of time. The upper curve obeys equation (4.41), or equation (4.43) with \( |h_1| = 3.5 \text{ cm} \); the lower curve shows the effect of \( h_1 \) by taking it equal to zero.

An approximate superposition principle, is, however, possible (Parlange et al., 1995a, 1995b; Ross et al., 1995). These approaches are based on the study of Gardner (1958) that when equation (4.34) holds, the steady-state Richards’ equation can be linearized by using \( \int D \partial / \partial x \partial \) as the dependent variable. Even for unsteady flow, \( \partial / \partial t \) in equation (4.16) can be small for soils. This observation led us to equation (4.33), where the flux, \( q \), becomes independent of position, and where \( D \) is a delta-function. For normal soils, \( \partial / \partial t \) is small for most of the profile in equation (4.16) since \( D \) is frequently close to a delta-function. Thus, good approximations should be obtained by superposition of solutions when \( \int h / D \partial \partial x \partial \) is taken as the dependent variable. Here, \( f \) is any conveniently slowly varying function of \( \phi \), compared with \( D \). It is interesting that Richards’ equation can be linearized for both of the two limiting mathematical behaviors of \( D \): a constant \( D \) and a delta-function \( D \). For instance, consider the interaction of the profile given by equation (4.20), with an impervious wall located at \( x = L \). Clearly, a convenient dependent variable is \( f = \phi^{-1} \). As long as \( f D / \partial \partial x \partial \) is finite and the wetting front is at a finite distance, then the normalized moisture content at \( x \) and \( \phi(x, t) \) can be found from

\[
2 \int_0^t \frac{D}{\partial} \partial \partial \partial - \int_0^t \frac{D}{\partial} \partial \partial \partial = \frac{SL}{\sqrt{t}} + \frac{A}{2t} \partial \partial \partial^2 + \frac{A}{2}[L - x]^2
\]  
(4.47)

and for \( t > t_i \) given by

\[
2 \int_0^t D / \partial \partial \partial = SL / \sqrt{t_i} + \frac{1}{2} \partial \partial \partial^2 / t_i
\]  
(4.48)

where \( t_i \) is the time when interaction of wall and wetting profile first takes place. If \( t_i = 0 \), such that \( \int D / \partial \partial \partial \partial \partial \partial \) is infinite, then there is an insignificant “tail” ahead of the main profile. This can be eliminated, for instance by replacing \( D \) by \( D - D(\phi = 0) \) in
Figure 4.5 shows the value of \( q \) as a function of time, when \( L = 1 \). Comparison with Figure 4.6 gives \( \theta = 0 \).

\[
\frac{d - 1}{d - (\phi - 1)} = \phi
\]

(4.54)

The difficulty of solving for a flux of \( x \) per unit length of the source is shown in Figure 4.5, where the source is depicted as a point source.

\[
\int_{x=0}^{x=L} \int_{z=0}^{z=L} = \phi
\]

(4.55)

As an illustration, consider the case where \( \phi = 0 \).

\[
\int_{x=0}^{x=L} \int_{z=0}^{z=L} = \phi
\]

(4.56)

From equation (4.9), we find that \( q \) increases rapidly with \( \theta \).

\[
q = \theta
\]

(4.57)

Finally, the value of \( q \) when \( L = 1 \) is given by equation 4.9. Figure 4.1 shows that the solution is consistent with the boundary conditions.

\[
q = \theta
\]

(4.91)

From equation 4.9, we find that \( q \) increases rapidly with \( \theta \).

\[
q = \theta
\]

(4.92)

Figure 4.6 shows the value of \( q \) as a function of time, when \( L = 1 \). Comparison with Figure 4.5 gives \( \theta = 0 \).

\[
\frac{d - 1}{d - (\phi - 1)} = \phi
\]

(4.54)
Fingered Flow in Homogeneous Sandy Soils

Interest in subsurface transport mechanisms responsible for unusual behavior has increased with increasing concern over groundwater contamination by pesticides and nonaqueous liquids. One particular mechanism that is attracting attention is fingered flow due to instability of the wetting front.

Chuoke et al. (1959) tried to extend the Hele-Shaw cell result of Saffman and Taylor (1958) by introducing the curve-fitting parameter of the "effective surface tension." More fundamentally, Parlange and Hill (1976) looked at the balance between capillary and gravity forces in a diffuse wetting front as it penetrates into a coarse sand. The resulting description of fingered predicted the finger width and this was later improved by Baker and Hillel (1990), who discussed the effect of the water entry pressure. Glass et al. (1991) extended the theory to three-dimensional fingers, while Selker et al. (1992) analyzed the water distribution within a finger, and Liu et al. (1995) showed the importance of hysteresis in the formation of fingers.

In its simplest form, the width, \( d_a \), of the most unstable finger for an air–water system is given as (Parlange and Hill, 1976)

\[
d_a = \frac{nS_p^2}{K_p(\theta_f - \theta_i)}
\]  

(4.56)

where subscript "a" represents the displaced phase (air) and \( F \) refers to properties at the wettest point in the finger, namely the tip. Baker and Hillel (1990) pointed out that for a fully developed finger, "F" should refer to the water entry value. However, drier conditions can be observed (figure 4.7) as a result of hysteresis (Liu et al., 1994, 1995). In equation (4.56), two-dimensional fingers are considered. In three dimensions, the same expression holds for the diameter of the finger, but with \( \pi \) simply replaced by 4.8 (Glass et al., 1991).

Behind the wetting tip, the water content decreases, so that as a finger passes by a tensiometer the pressure increases rapidly following the passage of the tip when the tip reaches it, and then pressure decreases slowly as the tip progresses further on. The following equation was developed by Selker et al. (1992):

\[
z = \int_{h_{rel}}^{h_{rel}} \frac{dh}{[1 - \frac{v}{v}]}
\]  

(4.57)

where \( z \) is the distance measured upward from the finger tip and \( v \) is the constant finger velocity. As \( z \to \infty \), the pressure stabilizes so that \( v(\theta/K)_{rel} = 1 \). In practice, the pressure may stabilize at a higher value if the fingered flow ceases to exist. This progression is presented in figure 4.8.

Many new and interesting fingering phenomena arise when the fluids are oil/water, rather than air/water. In particular, if the oil phase is free to move, then...
Where $W$ is the initial moisture content of the soil and $W'$ is the moisture content of the soil after a period of time.

The equation for the change in moisture is given by:

$$ dW - dW' = dW $$

This equation holds true if the transpiration is greater than the number of pore groups. The maximum amount of moisture in each pore group can be calculated using:

$$ W = \frac{1}{N} \sum_{i=1}^{N} i \cdot 2^{i-1} $$

where $N$ is the total number of pore groups.

For preferential flow in the soil, the soil moisture content decreases with depth, leading to a decrease in the hydraulic gradient for water movement. This is depicted in Figure 4.9, where the water flow is shown in a cross-sectional view of the soil profile.}

In the lower sections, water movement is only downwards, while in the upper sections, water movement is a combination of both horizontal and vertical movement. The presence of preferential flow paths can significantly influence the moisture dynamics within the soil profile.
where the moisture content in each pore group, \( \theta_p(z, 0) \), is a given function of the depth when \( t = 0 \). Equation (4.65) can only be integrated easily if \( A_p \) is a simple function or just some constant. This last case is when there is no exchange of water between the pore groups. Similar expressions can be written for \( z < \beta vt \), which is controlled by the boundary condition at \( z = 0 \), rather than the initial condition as in equation (4.65).

For more complex situations, with realistic water exchange between pore groups, a numerical solution of equation (4.65) can be obtained by taking a fixed time step \( \Delta t \). Equation (4.64) shows that over time step \( \Delta t \), the moisture in class \( p \) will travel a distance \( \Delta z_p \) resulting in a solution at times \( t \) for the \( p \)th pore group:

\[
\theta_p(z, t) = \theta_p(z - \Delta z_p, t - \Delta t) + I_p, \quad \Delta z_p = \beta v_p \Delta t
\]

where \( I_p \) is the integral of \( A_p \) from \( (t - \Delta t) \) to \( t \). The numerical solution is simplified by choosing the ratio of the velocities [equation (4.63)] in two successive pore groups that are integer multiples. This is accomplished by introducing an appropriate piecewise linear conductivity function (figure 4.11). In the "Darcy flow region," it consists of a number of lines tangent to the conductivity function, with the slope of each consecutive tangent line always an integer multiple of the preceding line [equation (4.63)]; here, for instance, twice.

The procedure to find the limiting moisture contents of the pore groups from the unsaturated soil conductivity curve is given in Steenhuis and Parlange (1988). How the model works—and how it differs from models based on the Richards' equation—can be demonstrated with an example of steady-state saturated flow through a vertical column. The limiting moisture contents for each category of pores is shown schematically in figure 4.12. The moisture contents of the intersections of the conductivity function's linear pieces are illustrated in figure 4.11. For example, the limiting moisture contents for pores in the category, labeled \( p = 1 \), are 0.10 and 0.20 cm\(^3\)/cm\(^2\). Employing equation (4.63), we find that the velocity of flow for pores in categories \( p = 0, p = 1, p = 2 \), and \( p = 3 \) are 0, 1, 2, and 4 cm/h, respectively.

![Figure 4.11 Conductivity function for pores of varying moisture content as used in the preferential flow model.](image-url)
Figure 4.12 Velocity of water penetration as a function of moisture content. Pore groups correspond to linear pieces in figure 4.11. The preferential-flow model (left) shows that flow is faster through pores with greater moisture content. After 2 h, water in category $p=3$ pores has penetrated 8 cm below the surface; in contrast, convective-dispersive averaging (right) predicts penetration to only 3.5 cm.

Then, by using equation (4.66), we find, for example, that after 2 h, the water in the pores of category $p=0$ has not moved at all. The water in category $p=3$ pores has moved, by contrast, 8 cm. This is very different to a convective-dispersive analysis based on the average flow path, which would place the average depth of penetration at a uniform 3.5 cm, albeit with some minor dispersion around this depth.

The importance of modeling solute movement on the type of model used can also be shown by a series of experiments performed by Anderson and Bouma (1977a, 1977b) using undisturbed and well-structured soil cores. A 300-ppm chloride solution was applied at a rate of 1 cm/day as either a sudden pulse or at a steady-state rate. The resulting breakthrough curves were different, with the pulse application resulting in a faster response than the steady-state rate (figure 4.13a, b). We used the preferential-flow model to simulate this phenomenon, assuming that there was some exchange of the solutes between the flow paths. Additional details are given in either Steenhuis et al. (1990) or Stagnitti et al. (1994, 1995). The two simulations shown in figure 4.13 make different assumptions about the velocities in the pore groups. For simulation 1, the velocities in the nine pore groups increased by a factor of 2 from one group to the next larger pore group. In simulation 2, the only difference was that the velocity in the largest pore group was four times greater than in simulation 1. Figure 4.13 shows that the simulation model is capable of matching the breakthrough curve without changing any of the model parameters, except the input rate of chloride solution. In contrast, Anderson and Bouma (1977a, 1977b) tried to fit dispersion coefficients to the breakthrough curves by using the convective-dispersive equation. They found that the coefficients were highly dependent on the flow regime, indicating that the use of the convective-dispersive equation was not appropriate.

The different behavior of this preferential flow model and the Richards' equation can seem surprising because both models are based on Darcy's law and the continuity equation. But, Richards' equation is in some sense just a special case of the preferential-flow model. By choosing a function for "$A$" in equation (4.62) that assigns all moisture to the pore group with the smallest pore that is not filled up, a solution with a sharp wetting front can indeed be obtained with the preferential-flow model. However, the solution cannot be the same at early times because the preferential-flow model assumes a unit gradient. It cannot, therefore, represent the initial square root of time behavior. However, it is able to simulate the early arrival of invading water to the groundwater and the arrival of "passenger" chemicals with the first rainstorm after application.

**Water Movement—Large Scale**

In this review, which is based on the analysis of just a few problems, we have tried to present our current understanding of water transport processes in soils. However, there is still the need to connect our knowledge of local-scale processes to that of the modeling of watersheds. This connection is often tenuous or even trans-science, so empirical large-scale models are sometimes used in practice. However, as more processes are studied and become understood, such as preferential-flow paths, these large-scale models should become more realistic and physically based.

For instance, in humid, well-vegetated areas with shallow soils, such as in the northeastern United States, many no-till fields and permanent grasslands have a soil
SOIL PROPERTIES AND WATER MOVEMENT

Conclusions

In this chapter, we have mainly discussed the physical processes of water movement within and over the soil at the local scale that influence water movement at the watershed scale. Since 1955, when Davis (1955) found an empirical relationship between hydraulic head and flux, we have focused on describing the water movement needed for our mathematical models. In order to predict the water movement at the topography, soil types and soil water movement and subsurface structure, our ability to measure these parameters has improved tremendously; but measurement of subsurface characteristics has not kept up with our ability to describe the processes. Therefore, because of the lack of accurate subsurface data, the approaches to predicting water movement at a local scale have remained largely empirical. In addition, so as to have accurate information about surface processes, we may use a combination of topographic, soil moisture, and rainfall data, the approaches to predicting water movement at a local scale are likely to remain empirical for another 150 years of research.

References


(4.67)

where \( P_e \) is the effective rainfall and \( S_t \) is the maximum depth of water storage in the unsaturated zone. In practice, rainfall intensity can be neglected.


