Column Flow in Stratified Soils and Fingers in Hele-Shaw Cells: A Review

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We examine the development of instability studies in the oil industry and in soil physics. The former being far more advanced has tended to mold the latter. This had some unfortunate consequences as oil studies tended to rely heavily on Hele-Shaw cells which provide a poor model for soils. In soil physics creation of column flow in soils has serious implications for infiltration and the transport of pollutants. It is to John Philip’s credit that he recognized fairly early the practical importance of instability in soils.

INTRODUCTION

In his 1972 review [Philip, 1972] on “future problems of soil water research”, John Philip listed “Stability of soil water flows” as the first in a list of five topics. He stated “till now almost all analyses of water movement in unsaturated soils have contained the implicit assumption that solutions of the flow equation are stable for flows in homogeneous soils. This appears to be very reasonable since the diffusion character of the equation suggests that disturbances will be damped and not magnified; but it remains an open and interesting question whether the possibility of capillary hysteresis affects flow stability in homogeneous soils. Hill and Parlane have performed elegant experiments on instability (fingering) during infiltration into a soil with a fine-textured layer over a coarser-textured one...contact angle problems in a surface layer may well be explained as the expression of instability....This work is certain to stimulate further studies of stability...”

Even at that early stage, John Philip recognized the important implications of the work in modeling infiltration in the field as subsequently documented by Starr et al. [1978]. His statement on hysteresis was remarkably accurate as hysteresis is ubiquitous in fingering even though that recognition has been slow to develop. The interaction between contact angle problems and fingering remains a fundamental question and his suggestion that further studies would follow proved quite prophetic. Indeed shortly afterwards Raats [1973] and Bridge and Collis-George [1973] were the first to refer to Hill and Parlane in their respectively theoretical and experimental studies.

Philip [1972] also mentioned the earlier papers of Saffman and Taylor [1958] and Wooding [1969] which provide some theoretical background to fingering in stratified porous media as will be discussed later. In his important paper on fingering, Raats [1973] points out that “in almost all studies it has been tacitly assumed that small perturbations in flow patterns will tend to disappear. In other words, it is usually assumed that the flows are stable. Recent experiments by Hill and Parlane [1972] involving infiltration into layered soils suggest that this assumption is not always justified.”

EARLY WORK

It is curious that the practical importance of instability in stratified soils was not recognized prior to 1972. As stated
by Baker and Hillel [1990], "the phenomenon of fingering during infiltration into layered soils has attracted increasing interest since the work of Hill and Parlange [1972], who were among the first to demonstrate that fingering can occur in a fine-over coarse-textured profile". Most likely the earlier "demonstrations" had not obvious enough connection to infiltration in stratified soils, e.g. Horton and Rogers [1945]; Lapwood [1948]; Heller [1966]; Elder [1968]; and Bachmat and Elrick [1970] as they were concerned with instabilities caused by gradients of temperature or solute concentration in miscible liquids. Others were difficult to discover [Engelberts and Klinkenberg, 1951; DeRoo, 1952; and Tabuchi, 1961]. Tabuchi [1961] in particular drew a rough sketch of an embryonic finger starting to form in a layer of coarse glass beads underneath a layer of fine glass beads and obtained a necessary stability condition independently of Saffman and Taylor [1958]. Smith [1967] came close to our present understanding of fingering with "infiltration when the rate of rainfall is less than the infiltration capacity of the sand...For example, if a succession of drops from a burette falls upon a sand for several hours, it will be observed that these drops pass downwards in a filament which when established, does not spread. The balance of the sand is either dry or in an unsaturated condition...The problem arises largely in very light rainfall." The question of fingering in very light rainfall will be discussed briefly later on. Although not entirely clear, Crosby et al. [1968] referred to Smith's "ribbons" and possibly observed them in the case of a fine material overlying a coarse material..." and "this mechanism is the apparent explanation of the dry conditions found beneath the drain field in the Spokane Valley." An earlier paper by Miller and Gardner [1962] made similar points, but more emphasis was placed on the important point that "when most of the pores in a layer were larger than those in the surrounding soil, infiltration was temporarily inhibited...water must accumulate at a layer-soil interface until it is at a tension low enough to allow it to move into pores in the layer." Then "as the water passes through an initially dry sand layer, it characteristically wets the sand in only a few places and the remainder of the sand remains dry. Liquid movement through the sand is restricted to the water-filled channels." However the sand layers were only half a centimeter thick and eventually became uniformly wet.

Other papers put more emphasis on the presence of ribbons/tongues/fingers/columns, but as they were clearly caused by water repellency or air compressibility presumably these effects were seen as being of limited importance. For instance Bond [1964] states "rain penetrates into water repellent sands through narrow tongues, leaving the interven-
nomenon). Of course this does not mean that some of the equations in Saffman and Taylor [1958] and the similar ones in Chuoke et al. [1959] cannot be properly reinterpreted. For instance the general necessary condition for instability, for water and air, neglecting both viscosity and density of air becomes

\[ K > Q \] (1)

where \( Q \) is the water flux imposed by the upper layer of fine material, and \( K \) is some conductivity of water in the coarse layer underneath. Note that since conductivity is an increasing function of the water content, it is certainly necessary that

\[ K_s > Q \] (2)

where \( K_s \) has its maximum value at saturation. Clearly with a Hele-Shaw cell saturation is always imposed and there is no ambiguity as to the value of \( K_s \). Hill and Parlane [1972] also assumed that the fingers were saturated and thus took condition (2). Condition (2) is an obvious necessary condition since if \( Q \) was greater than \( K_s \) the whole area would have to be saturated to carry the water and no finger would be present. It is a fundamental contribution of Hillel [1987], Hillel and Baker [1988], and Baker and Hillel [1990] to have noticed that condition (2) is not constraining enough and condition (1) with (usually) a lower \( K \) must be satisfied. They associate this value with a “water entry” value for the water to penetrate the coarse layer. This condition provides a mechanism to explain the constriction of the water flow. Those points of entry were clearly observed by Glass et al. [1989]. Hillel and Baker [1988] further suggest that upon rewetting the water entry suction will be higher, resulting in drier fingers which was also observed by Glass et al. [1989] leading to interesting hysteresis phenomena [Liu et al., 1995; and Raats, 1973]. For instance hysteresis is crucial by limiting lateral capillary diffusion of water which would otherwise remove the presence of fingers.

Glass et al. [1989] further observed that there were too many points of entry for all of them to become fingers. Then, merger takes place until only a few fingers with an optimal size, remain. These mergers took place just below the interface of the layers in the original experiments of Hill and Parlane [1972] and could not be observed. Because of a more homogeneous packing the merger in Glass et al. [1989] took place over a greater distance and could be clearly observed. The region where mergers take place is called the induction zone [Hill and Parlane, 1972].

Saffman and Taylor [1958] predicted the optimal width of the fingers in a Hele-Shaw cell by balancing the destabilizing effects of gravity and the stabilizing effect of surface tension. The same formula was then “extended” by Chuoke et al. [1959] for a porous medium replacing the surface tension which loses its meaning for a diffuse wetting front by an “effective” surface tension. Well aware of this limitation, Raats [1973] and Philip [1975a] limited themselves to “Green and Ampt soils” with discontinuous wetting fronts and extended condition (1) for other situations, e.g. with nonwetting soils and compression of the air below the invading water.

It is interesting that Raats [1973] suggests that instability will take place for nonponding rainfall and a homogeneous, i.e. not stratified, soil, whereas Philip [1975a] does not. Raats [1973] states “infiltration of nonponding rainfall is very similar to infiltration of ponded water through a fine layer or crust as observed by Hill and Parlane [1972]”. Indeed following this similarity one must look for some equivalent mechanism resulting in the concentration of flow provided by the points of entry in layered soils. One might speculate that when a large raindrop (large compared to the pore size) hits the soil surface with a positive pressure it will enter primarily through the largest pores and passages and if enough raindrops fall in the same neighborhood they might merge fast enough to form a finger? This could possibly explain why at low rainfall rates when raindrops are less likely to merge, fingers become rapidly wider and eventually the instability disappears [Yao and Hendrickx, 1996]. Another possible mechanism might be linked to some wettability effects. However, Selker et al. [1992a,b,c] and Yao and Hendrickx [1996] observed fingers under nonponding rainfall and wetting sand, although wetting problems may well have affected field and laboratory observations, e.g. Hendrickx and Yao [1996] and Selker and Schroth [1998]. As discussed by Bond [1964] we expect water repellant soils to exhibit fingering, see also Bauters et al. [1998, 2000]. Note that oil displaced by water will tend to leave some oily residue on the sand grains leading to contact angle problems [Rimmer et al., 1996] which may well affect the oil flow [DiCarlo et al., 1997, 2000; Darnault et al., 1998; Rimmer et al., 1998; and Chao et al., 2000]. Thus, as suggested by Philip [1972] contact angle effects should always be considered at least as a contributing factor in the formation of fingers, especially for homogeneous soils.

Philip [1975a,b] was well aware that the limitations of a Green and Ampt soil “cast some doubt on the relevance of the model” but it “has the great advantage of being amenable to stability analysis. Unfortunately, formulations based on the Richards equation...are less so”. Indeed he refers properly to the model as a “generalized Hele-Shaw cell” and suggested “stability studies of appropriate forms of the Richards equation”, although “a general attack promises to be very
difficult. Philip [1975b] then provided an estimate of the finger width, corrected later by White et al. [1976], who also presented some experimental results, again mostly with a Hele-Shaw cell, because "it satisfies the criteria of the delta-function model precisely, whereas a soil water system can at best approximate them". They also observed fingering with a coarse sand by increasing the air pressure ahead of the wetting front. They concluded the study with further doubt on the delta-function model and the need of more "work on the stability of actual diffuse fronts", as done by Parlane and Hill [1976].

**ANALYSIS**

Parlane and Hill [1976] derived an expression for the finger width, $d$, based on the analysis of Richards equation, i.e. a diffuse front, yielding

$$ d = \pi \frac{S^2}{K(\theta - \theta_0)} \frac{1}{1 - Q/K} $$

where $S$ is the sorptivity given by

$$ S^2 = \int_{\theta_0}^{\theta} D \left[ \theta - \theta_0 - \theta_0 \right] d\theta, $$

where $D$ is the soil-water diffusivity and $\theta_0$ is the initial water content, assumed small enough that the soil-water conductivity $K$ at $\theta_0$ is negligible compared to its value at $\theta$. The coefficient $\pi$ is obtained for a two-dimensional finger as often observed in the laboratory, in the field where fingers are axisymmetric (more or less) the coefficient $\pi$ should be replaced by 4.8 [Glass et al., 1991].

Initially Parlane and Hill [1976] assumed that $\theta$ in Eq. (3) corresponds to saturation. However fingers are rarely saturated in soils (as they are in a Hele-Shaw cell). In fact their water content varies with depth. It was shown by Selker et al. [1992a,b] that $\theta$ varies with depth (measured from the interface between layers) according to the equation

$$ \theta = \int_{\theta_{o}}^{\theta} \frac{D d\theta}{K - \nu \left[ \theta - \theta_0 \right]} $$

where $\nu$ is the constant downward speed of the fingers obtained after a short time, i.e. after all mergers have taken place and the fingers have reached a steady configuration.

$\theta_o$ is the value of $\theta$ at $z = 0$ and if we assume, following Hillel and Baker [1988], that the maximum value of $\theta$ corresponds to a water entry value $\theta_o$, then, Eq. (5) gives

$$ \nu t = \int_{\theta_o}^{\theta} \frac{D d\theta}{K - \nu \left[ \theta - \theta_0 \right]} $$

which gives $\theta_o(t)$ when $\nu$ and $\theta_o$ are known. In particular when $t \to \infty \theta_o$ approaches an asymptotic value $\theta_{o,\infty}$ with

$$ K(\theta - \theta_{o,\infty}) = \nu (\theta_{o,\infty} - \theta). $$

Note that between $\theta_o$ and $\theta_{o,\infty}$ all properties are measured on a drying curve of the matric potential. However, as the finger moves downwards within the sand there is a very narrow zone at the finger tip where the water content increases rapidly, thus operating on a wetting curve but reliable matric potential data are impossible to get in that region [Liu et al., 1995; and Selker et al., 1992a,b].

Going back to Eq. (3) it is not entirely clear which value of $\theta$ should be used to determine $S$ and $K$. As noted by Hillel and Baker [1988], in agreement with Eqs. (1) and (3), we require $1 > Q/K$ or the total soil cross-section would be required to carry the water. This is true whenever $\theta_o < \theta \leq \theta_e$ is used. Indeed for steady state conditions the wetted fraction of soil is $F_{w} = Q/K_{e,\infty}$. This might suggest using $\theta = \theta_{o,\infty}$ in Eq. (3) however the fingers reach their thickness when $\theta_o$ is somewhat above $\theta_{o,\infty}$, but certainly less than $\theta_e$. When $\theta_o = \theta_e$ as already mentioned the fingers are very narrow and mergers are required before $d$ is obtained. Call $d_o$ this (unknown) value of $\theta_o$ to use in Eq. (3) and $K_o$ the corresponding value of $K$.

First if $F^{*} = Q/K_o$ is much less than one, $Q/K$ can be neglected in Eq. (3). Glass et al. [1991] looked at the impact of $[1 - Q/K]^{-1}$ on the value of $d$. By the time this is significant so many fingers are present that the impact of fingering, i.e. the fact that the flow bypasses most of the soil is lost. Thus for fingering to be important in practice, we require that $Q/K$ be negligible in Eq. (3). Then we obtain a simpler equation for $d$,

$$ d^* = \pi S_o^{*2} / K_o^{*} \left[ \theta^*_o - \theta^*_0 \right] $$

Note that both $S_o^{*2}$ and $K_o^{*}$ are inversely proportional to the viscosity, thus $d^*$ is independent of viscosity. The influence of viscosity can be felt only through $[1 - Q/K_o^{*}]$ and thus is irrelevant when fingering is important and $F^*$ is small.

We are now going to show that $d^*$ has only a very small dependence on $\theta_o$. For coarse sands and as long as $\theta$ is not too close to zero and to $\theta_o$, Parlane and Hogarth [1985]
petroleum studies and in soil physics. That connection was strongly influenced through the Hele-Shaw cell analogue, which turns out to have been at least misleading, both theoretically and experimentally, as fingers in the cells and column flow in stratified soils have very different properties and appearances. The presence of a fine textured layer over a coarse textured layer readily explains the appearance of points of entry at their interface resulting in column flow. In the case of a homogeneous soil the reasons for the concentration of flow are more speculative.

REFERENCES


