A simple model for predicting water table fluctuations in a tidal marsh

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[1] Wetland restoration efforts are ongoing in many urban estuaries. In this context the hydrologic characteristics of restored wetlands are of paramount importance since the spatially and temporally variable position of the water table and of soil saturation establishes the oxidation state of the substrate, which, in turn, affects the wetland’s biogeochemical composition and the biological communities it is capable of supporting. A relatively simple analytical model developed here describes tidal marsh hydrology from creek bank to interior, considering transient drainage, net meteorological inputs, and tidal effects. Given a series of physical and time-dependent inputs, the analytical solution derived predicts the position of the water table at points along a transect perpendicular to a tidal creek. Validation of the model using water table time series data collected along three transects at Piermont Marsh, a tidal wetland on the Hudson River in the New York/New Jersey Estuary, indicates good general agreement between observations and predictions, although it may not be precise enough for some kinds of ecological applications. A sensitivity analysis on the model indicates that a range of pairs of transmissivity and specific yield values that increase with distance from the creek results in the same spatial and temporal fluctuations in the water table. This equifinality result is discussed as it relates to the predictive capacity of the model presented.


1. Introduction

[2] In urban estuaries, filling, dredging, damming, and bulkheading have historically resulted in dramatic alteration of estuarine hydrology, with significant impacts to the functioning of wetlands [Mitsch and Gosselink, 2000]. To mitigate for wetland functional impairment, restoration, creation, and enhancement efforts seek to reestablish self-perpetuating ecosystems with hydrologic regimes typical of the surrounding region [Middleton, 1999]. One way to improve the success of these efforts, is through study of reference marshes. In particular, modeling of the hydrological processes occurring in relatively undisturbed reference marshes can be helpful in assessing the impacts of ongoing estuarine modifications on existing wetlands, and also in prioritizing, planning, and designing regional wetland improvement efforts [Niedowski, 2000; Shisler, 1990; Zedler, 2001].

[3] In tidal areas, hydrology at least partially controls the exchange of nutrients, organic matter, and pollutants between upland watersheds, tidal wetlands and surface waters [Gardner, 1975; Heinle and Flemer, 1976; Váliela et al., 1978; Luther et al., 1982; Hemond et al., 1984; Jordan and Correll, 1985; Yelverton and Hackney, 1986]. Recent hydrologic research in tidal wetlands has involved the development of theoretical models suggesting linkages between tidally and meteorologically driven groundwater flows and both soil aeration rates [Ursino et al., 2004; Li et al., 2005], and vegetation patterns [Silvestri and Marani, 2004], emphasizing the importance of groundwater flows on tidal marsh ecology [Wilson and Gardner, 2005; Marani et al., 2005]. In particular, the spatially and temporally variable position of the water table establishes soil saturation patterns, which, in turn, affect the oxidation state and biogeochemical composition of the substrate, as well as the biological communities the wetland is capable of supporting. Together these factors determine many aspects of tidal marsh ecohydrology.

[4] Although a variety of wetland hydrology models have been attempted, comparison of these attempts is complicated by intermarsh variability, the application of different modeling methodologies, and inconsistency in the focus of the modeling efforts. The most basic attempts [Youngs, 1965; Youngs et al., 1989] involve solution of basic drainage equations with unsteady water tables assumed to behave as a continuous succession of steady states. Application of the classic, nonsteady state drainage models (described by Ritzema [1994]) to wetland hydrology was not found in the literature. Some [Parlange et al., 1984; Nielsen, 1990; Barry et al., 1996; Li et al., 2000a, 2000b, 2002; Li and Jiao, 2003] have used the well-known Boussinesq equation to model the transient position of the water table in a shallow, rigid aquifer in contact with a sinusoidally oscillating reservoir. Jeng et al. [2005] develop an analytical solution to the Boussinesq equation to address spring-neap...
tide induced fluctuations in a sloping coastal aquifer. More
detailed hydrologic models have been derived to describe
various individual components of tidal marsh hydrology,
such as the vertical flow of groundwater in response to
evapotranspiration demand and the piezometric pressure of
an underlying aquifer [Hemond and Fifield, 1982], surface
infiltration [Hemond et al., 1984], horizontal fluxes [Nuttle
and Hemond, 1988], and the stress and pressure changes
due to tidal loading of the marsh surface [Reeves et al.,
2000]. Attempts to link together different hydrologic pro-
cesses in wetland environments have been undertaken using
numerical models [Ursino et al., 2004; Wilson and Gardiner,
2005; Li et al., 2005; Skags et al., 2005; Thompson et al.,
2004; Twilley and Chen, 1998]. However, few attempts to
validate wetland hydrology models in actual wetland envi-
noments have actually been performed.

The validation process employs a much longer set of
observations, collected at a much finer time step.

2. Site Description and Methodology

[7] Piermont Marsh is an irregularly flooded, brackish
tidal marsh located approximately 40 km north of New York
City along the western bank of the Hudson River (Figure 1).
The vegetation of Piermont Marsh is a combination of
Spartina alterniflora, along the edges of some creeks and
the Hudson River coastline; invasive Phragmites australis,
occupying approximately 75% of the overall marsh surface;
and the typical mix of grasses, sedges, and cattails found
in high marsh environments along the northeastern coast of
the United States. Tidal fluctuations in this span of the Hudson
River are sinusoidal and semi-diurnal, with a tidal range
of approximately 1.1 m. Salinity is in the mesohaline range.

[8] Various data were collected at Piermont Marsh to
to characterize the site hydrology [Montalto et al., 2006] and
to validate the model. Tide gages were installed in the
creeks and Hudson River. The elevation of mean high water
in this span of the river is 0.83 m in the 1929 National
Geodetic Vertical Datum, (NGVD-29). Topographic survey-
ning, conducted using real-time kinematic GPS and a laser
plane unit, revealed a primarily flat marsh surface, at an
average overall elevation of 0.95 ± 0.05 m NGVD-29. A
soil core extracted from the interior of Piermont Marsh
uncovered peat extending from the surface to a depth of 3 m,
below which an organic silt clay layer extends almost
completely uninterrupted to a depth of 9.5 m below the
surface. Basal sediments were not encountered in this, the
only soil core attempted in the marsh [Wong and Peteet,
1999]. The saturated hydraulic conductivity was measured
using the auger hole method. The average saturated hydra-
luric conductivity of the creek bank, 6.6 × 10⁻⁴ cm/s, is lower
than the overall marsh average, 7.75 × 10⁻³ cm/s [see
Montalto et al., 2006, Figure 3].

[9] The total depth of precipitation falling every 10 min
was measured continuously using a rain gauge. Daily
potential evapotranspiration (PET) rates were estimated by
the Northeast Regional Climate Center using MORECS, a
model employing the Penman-Monteith equation, and util-
izing data measured at White Plains, NY, approximately
10 km due east of Piermont Marsh.

[10] Pressure transducers were used to measure the posi-
tion of the water table elevation in a series of wells installed
along three different transects at Piermont Marsh (Figure 1).
Known as transects 2, 3, and 4, they are referred to in this
paper by the names used to describe them in Montalto et al.
[2006]. Wells were positioned along each transect at 6, 12,
18, 24, 36, and 48 m from the creek bank levee. Data
loggers were used to record the position of the water table
simultaneously for approximately one lunar month in
alls wells along each of the transects. Data was recorded
at 10-min intervals. Measurements were made from 24 April
to 22 May 1999 along transect 2, 25 May through
22 June 1999 along transect 3, and 28 November through
26 December 2000 along transect 4. Transects 2 and 3 were
located perpendicular to an approximately 4 m wide,
unnamed tidal creek located in the northern half of Piermont
Marsh. Transect 4 was situated perpendicular to the slightly
wider Crumkill Creek, located in the marsh center.
(Figure 1). Both creeks were less than 2 m deep at mean tide.

[11] A more complete description of the vegetation, substrate characteristics, tidal range, and hydrology of the site is provided elsewhere [Montalto et al., 2006]. A brief description of its hydrology follows. Figures 2, 3, and 4, adapted from Montalto et al. [2006], depict several weeks of water table observations in all wells along transect 2 in spring, transect 3 in summer, and transect 4 in winter conditions. The elevations are reported in height above the 0 m NGVD-29 datum. Different colors are used to designate the water table elevation at different well locations, with gray and dark blue used for the creek stage and levee well, respectively. Figures 2, 3, and 4 also display the zone of 95% confidence, a measure of the error incurred by pressure transducer calibration, which was in almost all cases, less than ±1 cm. Uncertainty of elevations surveyed using RTK GPS techniques accounts for another error of 0.5 to 2 cm. Thus, when pressure transducer noise is not a factor, the observed water table elevations may, in general, be considered accurate to within ±3 cm.

[12] As illustrated in Figures 2, 3, and 4, spring tides and other meteorologically induced high tides periodically inundate and saturate the surface of Piermont Marsh. Inundations along transect 2 occurred, for example on 1–3 May and then not again until 14–20 May. As described below and illustrated in Figure 5, immediately after the recession of inundating tides, a sloped water table is observed in wells located within 24 m of all tidal creeks. The water table is nearly flat and close to the surface further inland.

[13] In between inundations, water is lost from the marsh to both evapotranspiration and horizontal drainage to the creek. These losses cause the water table to drop throughout the marsh. Because surface inundation occurs several times each month, and losses are relatively small, the water table always remains within 10 cm of the surface, across the entire marsh interior (defined, for the purpose of this paper, as all areas located at least 36 m from creek banks), even...
throughout prolonged dry periods in the summer. The water table drops up to 30 cm below the marsh surface in portions of the marsh located closer to creek banks.

[14] Tidal fluctuations in the Hudson River and connected creek networks cause periodic, semidiurnal, and tidally induced fluctuations in the subsurface marsh water table. Fluctuations of up to 30 cm were observed in creek bank wells. Rarely were water table fluctuations in excess of 5 cm observed in wells located further than 6 m from the creek bank. No tidal fluctuations were noted further than 18 m from creek bank. [Montalto et al., 2006].

[15] More sporadic fluctuation of the water table was caused by noninundating high tides, such as at the levee and 6 m locations along transect 2 on 27 April, or at the levee, 6 m, and 12 m locations along transect 3 on 28 May. During these instances, the water table rose with the incoming tide, but then subsided much more slowly as the tide ebbed. We speculate that this hysteresis effect is caused when tidal water that entered the marsh aquifer through the large fiddler crab holes and other macropores is forced, subsequently, to drain out to the creek through the soil matrix. Other potential explanations include anisotropic hydraulic conductivity and nonuniform soil properties along the transect.

[16] Rainfall increased the water table elevation most prominently in the creek bank region, but did not appear to significantly alter the water table of the marsh interior (for example, see transect 2, on 8 May). This was likely the result of a larger specific yield in the organic rich marsh interior compared with the denser creek banks.

3. Model Development

[17] An analytical model was developed to simulate spatial and temporal water table fluctuations in an irregularly flooded tidal marsh, bounded by two, parallel creeks. The solution derived is symmetric about the marsh midpoint. Evapotranspiration and precipitation vary temporally but were assumed not to vary spatially across the marsh. The substrate was uniform with an inelastic soil matrix, and a uniform specific yield. A no flow boundary condition was located between the upper peat strata and the underlying organic, silty clay horizon.

[18] On the basis of observations of small differences in nested piezometer readings and increasing salinity with depth, groundwater upwelling appeared to be minimal. A downward gradient of 0.02 m/m was measured during and after spring tide inundations. A similar gradient in the opposite direction was noted during long dry periods between tide inundations. Measured salinity levels were 8–10 μg/L at the surface and 14–16 μg/L at 2.5 m depth. If groundwater upwelling were important the deeper piezometers would have demonstrated lower salt concentrations than at the surface.
Flow normal to the transects was ignored. Moreover, since horizontal distances were much larger than the depth of the aquifer, Dupuit assumptions were valid. Flow could therefore be modeled in one dimension perpendicular to the creek bank.

Because of both the small depth of the unsaturated zone and frequent inundations, we assumed that moisture content in the unsaturated zone remained constant and equal to the saturated moisture content minus the specific yield. The actual value of the saturated moisture content is not important though, since only specific yield appears in the equations.

In addition, inundation of the marsh surface as well as surface runoff are assumed to take place instantaneously. Since groundwater was only moderately saline (<16 μg/L NaCl), density flow effects were ignored.

The governing equation is the Boussinesq equation [Boussinesq, 1877] for unsteady flow in a phreatic aquifer with accretion. Given the above assumptions and linearizing (because changes in water table depth are small compared to the overall depth of groundwater, this equation can be written as follows:

\[ S_y \frac{\partial d}{\partial t} = w + T \frac{\partial^2 d}{\partial x^2} \]  

where \( S_y \) is the specific yield \([L^3/L^3]\), \( t \) is the time \([T]\), \( d \) is the elevation of the water table \([L]\), \( T \) is the aquifer transmissivity \([L^2/T]\), \( x \) is the coordinate perpendicular to the creek \([L]\) and \( w \) is the net accretion \([L/T]\). This equation is similar in form to the classical, one-dimensional, linear heat flow equation, for which many solutions have been derived [see Carslaw and Jaeger, 1959].

To solve equation (1) directly, a solution must be found that meets the following conditions:

\[ d(x = 0, t) = \sum_{i=1}^{N} A_i \cos(\omega_i t - \alpha_i) \]  

\[ \frac{\partial d(x = \frac{L}{2}, t)}{\partial x} = 0 \]  

\[ d(x, t = 0) = f(x). \]

The levee water table is used as the creekside flow boundary \((x = 0)\) to avoid the difficulty of modeling seepage face phenomena on the creek bank wall, and also to avoid the need to model radial flow from the creeks since the creek bed does not rest on the impermeable lower boundary to flow. Regarding the latter, radial flow would invalidate...
the Dupuit assumptions. According to Hooghoudt [1940], radial flow occurs within a distance equal to \( D/\sqrt{2} \) back from the middle of a partial penetrating creek. At this site, the aquifer thickness, \( D \), is approximately equal to 3 m, which, as noted before, is the distance between the average water level in the levee well and the boundary of the highly permeable peat and the organic silty clay layer horizon with a low conductivity. To ensure validity of the Boussinesq equation, therefore, the boundary condition needed to be located at least 2.1 m from the center of the creek, a condition met by selecting the creek bank levee since both creeks were wider than 4 m.

Equation (2) is the periodic boundary condition at the levee well due to the tides with \( A_i, \omega_i \) and \( \alpha_i \) representing the amplitude, angular velocity, and phase angles, respectively as a Fourier series. Equation (3) is the “no-flux” condition at the marsh midpoint. The function, \( f(x) \), in equation (4) describes the initial water table profile, defined as the water table profile observed in the marsh immediately after the ebbing of an inundating tide. Although it is not general, a linear, sloping segment (Figure 5) has been shown to represent reasonably and sufficiently the post-inundation water table profile in the creek bank region [Montalto et al., 2006]. Accordingly, it is defined as follows:

\[
\begin{align*}
  f(x) &= mx + b & 0 < x < x_1 \\
  &= mx_1 + b & x_1 < x < L/2
\end{align*}
\]

where \( L \) is the distance between two tide creeks, as illustrated in Figure 6, and \( x_1 \), \( b \), and \( m \) are fitting parameters. 

Because equation (1) is linear, the original problem can be decomposed into three, individually modeled processes, the solutions of which are added, yielding a time and space dependent solution for the position of the marsh water table, \( d(x, t) \), vis-à-vis:

\[
d(x, t) = d_1(x, t) + d_2(x, t) + d_3(x, t) + d_{\text{avg}}
\]

where \( d_1(x, t) \), \( d_2(x, t) \) and \( d_3(x, t) \) represent the individual contribution to water table height of respectively the following processes: horizontal drainage flow to the creek (process 1 shown in Figure 6, top); the net effect of precipitation and evapotranspiration on the water table
(process 2 shown in Figure 6, middle); and the tidally induced oscillations of the marsh water table (process 3 shown in Figure 6, bottom). The $d_{avg}$ is used so that the water table can be given in height above the NGVD-29 datum. The boundary and initial conditions for individual water table heights are determined such that $d(x,t)$ in equation (6) meets the conditions set forth in equations (2)–(5).

Each of these three processes with boundary and initial conditions and the computer program used to run the model simulation is described separately below. A typical model simulation started at the beginning of the observations ($t = 0$), and continued through the end of the observation period, a period that spanned at least one full lunar month. Each inundation event during the observation period reset the initial conditions, as will be described in the following.

3.1. Process 1: Horizontal Drainage

[28] The water table profile established in the marsh upon recession of inundating tidal waters is similar after each

Figure 5. Immediate post inundation water table profile ($t = 0$) used for the initial conditions for flow process 1 along (top) transect 2, (middle) transect 3, and (bottom) transect 4. It was derived from well observations made after several spring tide inundation events and is represented by two line segments (equation (9)).

Figure 6. Graphical illustration of the three processes (not to scale). (top) Process 1, which simulates horizontal sideways drainage toward the creek, (middle) process 2, which simulates the effects of meteorological fluxes across the marsh surface, and (bottom) process 3, which models the propagation of tidal pulses in the aquifer.
flooding event, and decays subsequently as a result of horizontal drainage to the creek. Meteorological fluxes through the marsh surface are not considered in this process. Equation (1) is solved by separation of variables with the following conditions:

\[ d_1(x, t) = 0 \quad (7) \]

\[ \frac{\partial d_1(x = \frac{L}{2}, t)}{\partial x} = 0 \quad (8) \]

\[ d_1(x, t = 0) = f(x) \quad \text{where} \quad f(x) = mx + b \quad 0 < x < x_1 \]

\[ f(x) = mx_1 + b \quad x_1 < x < L/2 \quad (9) \]

The solution is written as

\[ d_1(x, t) = \sum_{n=0}^{\infty} C_n \exp \left( \frac{-(2n + 1)^2 \pi^2 t}{L^2} \right) \sin \left( \frac{(2n + 1)\pi x}{L} \right) \quad (10a) \]

where,

\[ C_n = \frac{4}{(2n + 1)\pi} \left[ b + \frac{mL}{(2n + 1)\pi} \sin \left( \frac{(2n + 1)\pi x_1}{L} \right) \right] \quad (10b) \]

\( x \) is the distance from the levee well and all other parameters are as defined previously.

### 3.2. Process 2: Meteorological Effects

[29] This solution quantifies the net effect of evapotranspiration and precipitation on the marsh water table profile. The Kraijenhoff van de Leur [1958] solution did not converge for the small time steps used here and, consequently, a more cumbersome two-step procedure is used. The solution needed to meet the following conditions:

\[ d_2(x, t = 0) = 0 \quad (11) \]

\[ \frac{\partial d_2(x = \frac{L}{2}, t)}{\partial x} = 0 \quad (12) \]

\[ d_2(x, t = 0) = 0. \quad (13) \]

In the first step of the solution, the net meteorological flux is initially assumed to act uniformly over the whole transect water table. The second step involves adjusting the solution once a day to satisfy equation (11). A daily adjustment is considered appropriate because the small error introduced during part of the day and the additional computational burden when more frequent adjustments are made. The equations derived for each of these two steps are elaborated below. Their usage in the computer simulation is described in the section 3.4.

[30] 1. To compute the uniform effects of precipitation and evapotranspiration, the second term on the right side of equation (1) is set equal to zero, because a uniform increase or decrease in the elevation of the water table does not result in horizontal flow. Then, separating variables and integrating, we obtain

\[ d_{2w}(t) = d_{2w}(t = 0) + \int^{t}_{0} \frac{w(t)}{S_p} dt \quad (14) \]

where \( d_{2w}(t) \) is the cumulative sum of all individual net meteorological fluxes through time, \( t \), and \( w(t) \) is equal to the precipitation, \( P(t) \), minus the total potential evapotranspiration, \( PET(t) \), at time, \( t \). The potential, and not actual, evapotranspiration values were used because the marsh substrate, even above the water table, was almost always near saturation during the observation period.

[31] 2. Equation (14) does not meet the boundary condition at \( x = 0 \). To adjust the solution so that it meets the boundary condition imposed by equation (11), the amount of water gained and lost through the creek bank due to evaporation and rainfall is calculated by solving equation (1) by separation of variables, to satisfy equation (11), equation (12), and the following:

\[ d_{2c}(x, t = t_j) = h_{oj} \quad (15) \]

where \( h_{oj} \) equals the total daily precipitation minus the total daily evapotranspiration on day \( j \). This operation yields the Glover Dumm equation [Dumm, 1954; Ritzema, 1994]:

\[ d_{2oj}(x, t) = \frac{4h_{oj}}{\pi} \sum_{n=0}^{\infty} \frac{1}{2n + 1} \sin \left( \frac{(2n + 1)\pi x}{L} \right) \exp \left( \frac{-(2n + 1)^2 \pi^2 (t - t_j)}{L^2} \right) \quad (16) \]

where \( d_{2o}(x, t) \) represents the position of the transect water table as a result of all net meteorological fluxes through the marsh surface and all associated horizontal movement of water through the creek bank, occurring during a given day; \( t \) is the time elapsed since the beginning of the current interval of the simulation; and \( t_j \) is the time elapsed between the beginning of the current interval of the simulation and the beginning of day \( j \) (for example: \( t_j = 0 \) when \( j = 1 \)).

### 3.3. Process 3: Tidal Effects

[32] This process simulates the propagation of creek stage fluctuations in the marsh aquifer. The tidal fluctuations observed in the creek bank levee well are used as boundary condition.

[33] Many solutions to this problem have been derived (Steggeventz, 1933; Todd, 1980; Knight, 1981; Parlange et al., 1984; Nuttle, 1988; Hughes et al., 1998; Sun, 1997; Li et al., 2000a, 2000b, 2002; Li and Jiao, 2003). The solution derived here follows the linearized approach presented by Sun [1997]. It involves solving equation (1) with \( w = 0 \) and according to the following conditions:

\[ d_3(x = 0, t) = \sum_{i=1}^{m} A_i \cos(\omega_i t - \alpha_i) \quad (17) \]

\[ \frac{\partial d_3(x = \frac{L}{2}, t)}{\partial x} = 0 \quad (18) \]

\[ d_3(x, t = 0) = 0. \quad (19) \]
The boundary condition expressed by equation (19) cannot be met but, as discussed later, the error associated with this approximation is small.

As mentioned previously, the water table fluctuations in the levee well are converted into a cosine series for use as the boundary condition at \( x = 0 \) (equation (17)). Equation (1) is solved by writing the boundary condition in equation (17) as the real part of an exponential function and by then assuming a similar form for \( d_3(x, t) \) that meets the boundary conditions. The resulting solution is the “tidal solution” depicted graphically in Figure 6 (bottom):

\[
d_3(x, t) = \sum_{j=1}^{m} A_j e^{-x/c_0} \cos \left( -\sqrt{\frac{\omega_j S_y}{2T}} x + \omega_j t - \alpha_j \right). \tag{20}
\]

The value of \( d_3(x, t) \) per equation (20) decreases exponentially with increasing \( x \), limiting the distance inland that tidally induced fluctuations in the water table can be transmitted. Assuming tidal characteristics, transmissivity, and specific yield values typical for this kind of site, we can expect \( d_3(x, t) \rightarrow 0 \) for \( x > 20 \) m. Therefore the boundary condition in equation (18) is satisfied for all three Piermont Marsh transects. The value of \( d_3(x, t = 0) \neq 0 \) as it should per equation (19). However, this value also decreases exponentially with \( x \). As will be discussed below, the error it causes is negligible, because its magnitude for all \( x > 0 \) will always be smaller than the amplitude of the tidal fluctuations considered.

### 3.4. Model Simulation

A computer program was developed to run the model. Using the creek bank levee well observations as the boundary at \( x = 0 \), water table fluctuations at 6, 12, 18, 24, 36, and 48 m along transects 2–4 were simulated. The 10-min time step of the simulation was equal to the interval over which water table observations and precipitation measurements were made. The total daily PET values were converted into 10-min incremental values by distributing them between sunrise and sunset using a sinusoidal function. On each 10-min time step of the simulation, \( d_1, d_2, \) and \( d_3 \) were computed individually for each distance, \( x \), from the creek bank levee well being simulated. The predicted water table position, \( d(x, t) \), was the superposition of \( d_1, d_2, \) and \( d_3 \), plus the average elevation of the levee boundary condition, \( d_{avg} \), for that interval as per equation (6).

On any time step, \( t_k \), during which the elevation of the boundary condition at \( x = 0 \), i.e., \( d_3(x = 0, t_k) + d_{avg} \) in...
process 3, was found to be greater than the average marsh surface elevation along the transect, the marsh was assumed inundated, and \( d(x, t) \) was set equal to the water level at \( x = 0 \). Then when the elevation of the boundary condition at \( x = 0 \) fell below the average transect marsh surface elevation, time was reset to zero, all initial and boundary conditions reset for the three processes, and a new \( d_{avg} \) and set of harmonic constants in equation (17) were computed for the \( x = 0 \) process 3 boundary condition for the time interval ending at the next surface inundation event. Extensive amounts of precipitation also triggered the resetting of the initial conditions if more than three of the water table predictions made along the transect exceeded the elevation of the marsh surface.

If along the transect, \( d(x, t) \) per equation (6), was less than the quantity \( d_3(x = 0, t) + d_{avg} \) (the elevation of the levee water table), and \( d(x, t) \) was also less than the elevation of the average marsh surface elevation, then \( d(x, t) \) was set equal to the elevation of the boundary condition at \( x = 0 \). This stipulation was an ad hoc attempt to model the effect of fast drainage through preferential flow pathways that extend from the creek bank into the marsh substrate.

At the beginning of each time interval between the inundation events, the computer program computed the average value of the levee water table, \( d_{avg} \), for that interval. The program also used Fourier analysis to derive all the harmonic constants, \( A_i \), \( \omega_i \), and \( \alpha_i \), in equation (17) to represent the levee well data on that interval.

We will treat in detail here how we calculated \( d_2 \) since the computational steps were cumbersome. Equations (14) and (16) were used to compute \( d_2 \) as follows: Starting on day 1 for each 10 min time step \( d_2(x, 0 < t < 1) \) was computed directly from equation (14). At the end of the day, equation (16) was used to reconcile any deviations from the prescribed height at \( x = 0 \), as described earlier, with \( h_{o1} \) set equal to the cumulative sum of all 10-min \( w(t) \) values occurring on day 1. The simulation then continued by augmenting \( d_2 \) stepwise, by the amount, \( d_2(x, 1 < t < 2) \), such that \( d_2(x, 1 < t < 2) = d_2(x, t - t_1 = 1) + d_2(x, 1 < t < 2) \), with the history of \( d_2 \) extending back to the beginning of day 2. At \( t = 2 \) days, \( d_2(x, t = 2) \) is again calculated directly, this time by solving equation (16) twice, considering that two days have passed since the effects of \( h_{o1} \) were initiated, and one day has passed since the onset of \( h_{o2} \) and so on.

Several different sets of simulations were performed. In the first simulation set, (the initial “model runs,”) input parameters were selected based on values measured at the site or derived from the literature. The second set of simulations constituted a sensitivity analysis on the model, for which selected input parameters were varied above and below the initial “model run” values, as described in the

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**Figure 8.** Observed and predicted water table elevations at (a) 6, (b) 12, (c) 18, (d) 24, (e) 36, and (f) 48 m from the creek bank along transect 3. The y axis is in meters above the zero point on the NGVD-29 datum. The average marsh surface elevation along transect 3 is 1.03 m. The parameter set values are given in Table 3.
Finally, the model was run with the input parameter sets that the sensitivity analysis suggested would lead to the lowest error, and the results compared.

3.5. Estimation of Model Input Parameters

To run the model, input parameters and boundary conditions needed to be specified. Because of the precision of the surveying techniques employed, the marsh width, and average marsh surface elevation were easily measured. The Fourier analysis used to approximate the levee water table with a cosine series also produced negligible errors. The fitting parameters used to approximate the process 1 initial condition water table profile were also easily and accurately estimated based on field observations (Figure 5). The boundary condition at the midpoints between the creeks for transects 2, 3, and 4 were at 269 m, 188 m, and 559 m, respectively.

Estimation of the specific yield and substrate transmissivity was more difficult. The specific yield is a particularly difficult property to measure, and can demonstrate considerable spatial variability in the landscape [Ritzema, 1994]. The transmissivity is the product of the depth of the aquifer and the saturated hydraulic conductivity, both of which are not known with a high degree of accuracy. For these reasons, we opted to estimate these parameters for the initial “model run”. In the subsequent sensitivity analysis on the model, we varied them independently and systematically to test our initial assumptions and also to determine which pairs of specific yield and transmissivity values yielded reasonable predictions along the three different transects. The procedure employed in selecting the initial “model run” parameters and sensitivity ranges for these values is described below.

The initial “model run” estimate for the specific yield was based on literature values. Specific yield values following. Finally, the model was run with the input parameter sets that the sensitivity analysis suggested would lead to the lowest error, and the results compared.

Figure 9. Observed and predicted water table elevations at (a) 6, (b) 12, (c) 18, (d) 24, (e) 36, and (f) 48 m from the creek bank along transect 4. The y axis is in meters above the zero point on the NGVD-29 datum. The average marsh surface elevation along transect 4 is 1.01 m. The parameter set values are given in Table 3.
Montalto et al. estimated the specific yield associated with varying degrees of peat stratification, with higher values indicative of undecomposed sphagnum mosses. Because the water table fluctuations in Pierron Marsh occurred in the largely undecomposed, peaty root zone, they estimated a specific yield of 0.6 for use in the initial “model run” predictions. For the sensitivity analysis, the specific yield was varied at 0.02 unit intervals from 0.02 to 0.87 for all transects. The higher-resolution measurements for peat in the literature (see summary by Price and Schlotzhauer [1999]) also reported for peat in the literature range from 0.09 to 0.84, with higher values more typical of undecomposed or living peat, and lower values more representative of decomposed strata [Price and Schlotzhauer, 1999]. Boelter [1965] also measured the specific yield associated with varying degrees of decomposition for peat samples taken in Minnesota bogs and reported values of 0.52–0.79 for the top 25 cm of undecomposed sphagnum mosses. Because the water table fluctuations in Pierron Marsh occurred in the largely undecomposed, peaty root zone, we estimated a specific yield of 0.6 for use in the initial “model run” predictions along the three transects. For the sensitivity analysis, the specific yield was varied at 0.02 unit intervals from 0.02 to 0.98. Although this range likely includes values that were too high and too low to describe the peat at Pierron Marsh, they represent the entire range of specific yield values reported for peat in the literature.

Estimates of the aquifer depth and the saturated hydraulic conductivity are needed to calculate transmissivity. Assuming the top of the organic silty clay layer as an impermeable boundary to flow, an aquifer depth of 3 m was assumed. Because the saturated hydraulic conductivity values measured in the creek bank substrate were lower than those measured more internally [Montalto et al., 2006], it was assumed that the creek bank rates control horizontal flux. $K_s$ values measured in the creek banks of transects 2 and 3, were $0.46 \times 10^{-3}$ and $0.87 \times 10^{-3}$ cm/s, respectively (no measurements were made along transect 4). The average of these two values, $6.6 \times 10^{-4}$ cm/s, multiplied by the 3 m effective substrate depth, results in an estimated transmissivity of $1.7 \times 10^{-2}$ cm²/d, the value used in the initial “model run” for each transect.

In the sensitivity analysis the transmissivity values were varied to reflect a range of $K_s$ values for marsh peat found in the literature (see summary by Montalto et al. [2006]) and also a range of effective depths extending from 1 m to the 12 m deep core extracted by Wong and Peteet [1999]. $T$ was varied on 1 m to the 12 m deep core extracted by Wong and Peteet [1999]. $T$ was varied on 1 m to the 12 m deep core extracted by Wong and Peteet [1999].

### Table 1. Variability of Standard Error, Mean Deviation, Mean Cumulative Error, and Maximum Deviation With Different Pairs of Transmissivity and Specific Yield at Each Distance on Each Transect

<table>
<thead>
<tr>
<th>Distance, m</th>
<th>Transect 2</th>
<th>Transect 3</th>
<th>Transect 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SE, cm</td>
<td>MD, cm</td>
<td>MCE, cm</td>
</tr>
<tr>
<td>6</td>
<td>m.r.</td>
<td>6.5</td>
<td>5.6</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>2.9</td>
<td>2.5</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>2.4</td>
<td>2.0</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>2.4</td>
<td>1.9</td>
</tr>
<tr>
<td>12</td>
<td>m.r.</td>
<td>5.6</td>
<td>4.5</td>
</tr>
<tr>
<td>12</td>
<td>1</td>
<td>3.2</td>
<td>2.6</td>
</tr>
<tr>
<td>12</td>
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<td>3.2</td>
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<tr>
<td>12</td>
<td>3</td>
<td>3.8</td>
<td>2.9</td>
</tr>
<tr>
<td>18</td>
<td>m.r.</td>
<td>4.5</td>
<td>3.6</td>
</tr>
<tr>
<td>18</td>
<td>1</td>
<td>2.2</td>
<td>1.8</td>
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<td>3</td>
<td>3.6</td>
<td>2.9</td>
</tr>
<tr>
<td>24</td>
<td>m.r.</td>
<td>4.2</td>
<td>3.7</td>
</tr>
<tr>
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<td>2.2</td>
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<td>1.9</td>
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<td>24</td>
<td>3</td>
<td>3.6</td>
<td>3.0</td>
</tr>
<tr>
<td>36</td>
<td>m.r.</td>
<td>2.6</td>
<td>2.4</td>
</tr>
<tr>
<td>36</td>
<td>1</td>
<td>2.4</td>
<td>2.0</td>
</tr>
<tr>
<td>36</td>
<td>2</td>
<td>1.6</td>
<td>1.4</td>
</tr>
<tr>
<td>36</td>
<td>3</td>
<td>2.4</td>
<td>2.0</td>
</tr>
</tbody>
</table>

### Table 2. Percent of All Timesteps During Which the Preferential Flow Approximation Was Employed

<table>
<thead>
<tr>
<th>Distance, m</th>
<th>Transect 2</th>
<th>Transect 3</th>
<th>Transect 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>6.5</td>
<td>7.6</td>
<td>4.1</td>
</tr>
<tr>
<td>12</td>
<td>2.5</td>
<td>4.1</td>
<td>2.1</td>
</tr>
<tr>
<td>18</td>
<td>1.1</td>
<td>2.1</td>
<td>1.2</td>
</tr>
<tr>
<td>24</td>
<td>0.2</td>
<td>0.4</td>
<td>0.5</td>
</tr>
<tr>
<td>36</td>
<td>0.2</td>
<td>0.2</td>
<td>0.5</td>
</tr>
<tr>
<td>48</td>
<td>0.2</td>
<td>0.2</td>
<td>0.5</td>
</tr>
</tbody>
</table>
Varying the specific yield and transmissivity incrementally within these ranges, 3626 simulation runs were made for each well location along each transect.

4. Results

To assess the closeness of fit of the predictions to the observations, and to assess whether the model has a tendency to overpredict or underpredict the water table elevation, three error calculations were computed: the standard error, the mean deviation, and the mean cumulative error. The formulas used to compute these are provided in equations (21)–(23):

\[ SE = \sqrt{\frac{\sum(o(x, t) - d(x, t))^2}{n - 2}} \]
\[ MD = \frac{\sum|o(x, t) - d(x, t)|}{n} \]
\[ MCE = \frac{\sum(o(x, t) - d(x, t))}{n} \]

where \( o(x, t) \) are the observations, \( d(x, t) \) are the predictions, \( n \) equals the number of observations in the simulation, and SE, MD, and MCE refer to the standard error, the mean deviation, and the mean cumulative error, respectively. The rationale for computing these three errors is as follows: The standard error and the mean deviation are both used to assess the model's general goodness of fit. The mean cumulative error allows tracking of the direction in which the model is biased. Also computed to aid in model validation, is the time step corresponding to the maximum absolute difference (MAX) between the predicted and observed water table elevations for each well in each simulation. This is used to identify when the model predictions deviate most from the observations.

4.1. Model Runs

The predicted water table heights for the “model runs” of transects 2, 3, and 4 are shown in Figures 7, 8, and 9. Table 1 lists the all the computed errors. The standard errors in the model run predictions were under 3 cm (the accuracy of the observations) for the 36 m well along transect 2, all wells located \( \geq 18 \) m from the creek bank along transect 3, and all wells \( \geq 12 \) m from the creek bank along transect 4. The remaining wells displayed slightly higher errors.

As expected, the computed mean deviations are slightly smaller than the standard errors computed, for a given simulation. The mean cumulative errors computed ranged from -4.9 cm to 0.55 cm. In all except the predictions made at the 6 m well location along transect 3, the mean cumulative errors were negative, indicating that
with the initial “model run” input parameters, the model
overestimates the position of the water table, along all three
transects. The simulations that most overpredict (and most
underpredict) the water table are predictions made at 6 m
locations, indicating greater overall bias in model predic-
tions at those locations.

The maximum deviation between observed and pre-
dicted water tables varied in the simulations from approx-
imately 6 to 21 cm, and corresponded usually with marsh
surface inundation. For example, all of the transect 3
maximum deviations occurred on 9 June between 5:30
and 6:30 PM. This is because the model assumes that the
whole marsh inundates instantaneously, while in reality the
inundation advances with the rising tide. In addition, large
deviations of the 6 m, 12 m, 18 m, and 24 m predictions
along transect 2 were caused by model underpredicting the
drawdown between inundation events.

Table 2 lists the number of time steps over which the
model invoked fast drainage through preferential flow paths
for each transect. At most, the preferential flow approxima-
tion was used on 7.6% of the predictions made on the 6 m
predictions along transect 3. It was used for less than 1–2%
of all the predictions made at the other well locations along
all of the transects.

4.2. Sensitivity Analysis

The purpose of the sensitivity analysis was to deter-
mine if different input parameter sets could improve the
model predictions. Figures 10–15 are contour diagrams of
the standard errors and mean cumulative errors, for sensi-
tivity analysis simulations. In Figures 10–15, the transmis-
sivity axis is on a log scale so as to facilitate the visibility of
results throughout the range of values tested. No standard
error contours are shown for the 48 m well along transect 2
because no observations were available for comparison due
to a pressure transducer malfunction at that location. We do
not show contour diagrams of the mean deviations, because
of their similarity to the standard error contours.

The standard error contour diagrams (Figures 10–12)
indicate that the “model run” input parameters do not
necessarily lie within the parameter regions that correspond
to minimum possible standard errors in the predictions at
the various well locations. For example, Figure 10 suggests
that an order of magnitude higher transmissivity than
estimated for the initial “model run” would generate better
predictions across the transect if the specific yield was held
at 0.6. Regions of <3 cm standard error (the accuracy of
the observations) can be found at almost all well locations
along all three transects. The only exceptions are the 6 m
and 12 m predictions along transect 3, where the minimum
possible standard errors are 4.1 cm and 3.7 cm, respectively.
Because the contours were generated at 1 cm intervals, the
minimum error contour shown on the charts, at the 6 m and
12 m locations, are 5 cm and 4 cm, respectively.

Along all three transects we observe that the range of
input parameter sets corresponding to reasonable (i.e.,

Figure 11. Results of the standard error sensitivity analysis for transect 3.
(≤3 cm) standard errors increases with distance from the creek. We also observe that in general, the greatest overall errors are in the region of low specific yield and high transmissivity.

[56] Figures 13–15 are mean cumulative error contours for all predictions made along the three transects. A 0 cm mean cumulative error contour appears on all the plots shown. As expected, high transmissivity values will tend to underestimate the position of the water table, because of increased lateral flow. The greatest overestimation of the position of the water table occurs at low to medium transmissivity values with medium to high specific yield. This is due to the fact that the lateral movement of water that entered the marsh substrate during inundation is inhibited by the transmissivity, while the higher specific yield values translate evapotranspirative losses into only minimal water table drawdown.

[57] From inspection of the data used to construct the contours in Figures 10–15, pairs of transmissivity and specific yield values that correspond to lower overall error across the transects were selected, for additional model runs. Three different pairs of Sy and T values were selected for each transect. These values are listed in Table 3.

[58] To visualize these model results, Figures 7–9 display the predicted water tables at each well location obtained by running the model with the T and Sy values listed in Table 3. Also shown in Figure 7–9 are the actual observations at each well location, as well as the initial “model run” predictions. Table 1 lists the standard error, mean deviation, mean cumulative error, and maximum deviation associated with each simulation along each transect.

[59] The predictions made with pairs of transmissivity and specific yield values selected from inspection of the sensitivity analysis fit the observations fairly well, and in general are better than the initial “model runs”. For example, at 2.3 cm, the average standard error across transect 2 using parameter set 2 is 50% of the standard error calculated using the “model run” parameter set (4.7 cm). The water table drawdown that occurs during the period of neap tides is well simulated, as is the effect of spring tide inundations on the water tables. It is interesting to note that in most cases, all three parameter pairs closely replicate these phenomena, even though the T and Sy values vary widely.

[60] As expected from the results of the sensitivity analysis, standard errors, and mean deviations of fewer than 3 cm (the error in the observations) were obtained for all simulations along all transects, with only the 12 m well along transect 2, and the 6 m and 12 m wells along transect 3 as exceptions. All of the mean cumulative errors obtained from the model runs made with the parameters listed in Table 3 were also within ±3 cm of the observations. Minor
differences in the maximum deviation from the model run were computed.

5. Discussion

This study indicates that the model, as presented, can be used to predict overall water table fluctuations in Piermont Marsh to within a reasonable degree of accuracy (i.e., to within ±3 cm of the observations). As an aid to identifying the optimum parameter spaces for each of the three transects, we have traced the parameter spaces corresponding to the overlap in the 3 cm error contours for the standard errors (Figure 16a), the mean cumulative error (Figure 16b), and the mean deviation (Figure 16c) values computed. To construct Figures 16a and 16b, in cases where the 3 cm contour was not available, the next lowest contour line was substituted. Notes in Figure 16 address specifics about this process. In Figure 16d, we have overlapped the standard error, mean deviation, and mean cumulative error optimum parameter regions, and indicated the position of the initial "model run" and Table 3 parameter sets, with little circles. The optimum mean deviation parameter space includes that of the standard error. It is therefore a less conservative measure of goodness of fit, and is for this reason not discussed further.

Figure 16 suggests that the initial specific yield and transmissivity values selected were reasonable for transects 3 and 4. An order of magnitude higher transmissivity would have yielded better predictions for the water table fluctuations along transect 2. Because of the physical proximity of transect 2 and 3, and their similar vegetative, topographic, morphological, and tidal characteristics, it is difficult to understand why this was the case. One possibility is that there is a difference in subsurface soil strata present in the two sites. Only additional field research can clarify this point. Another possibility is the different meteorological conditions witnessed during each of the two observation/simulation periods, as discussed below.

In Figure 17, the areas of the regions included within the 3 cm standard error contour (if it was available) for all simulations made along all transects are shown. In general, the parameter space corresponding to fewer than 3 cm standard error increases with distance from the creek bank. Similarly, and with only a few exceptions, the range of transmissivity values for which appropriate specific yield values will result in minimal standard error in the predictions, also increases with distance from the creek. These observations demonstrate decreasing importance of the transmissivity in determining the water table profile decreases with distance of creek. This is likely because of the greater overall complexity of water table dynamics in the creek bank zone over the marsh interior.

The shape of the optimum parameters spaces depicted in Figure 16 implies a wide range of transmissivity and specific yield pairs that will give the same predictions. The transect 3 space, for example, suggests that a marsh
characterized by a uniform substrate with a specific yield as low as 0.4 and a transmissivity below 0.01 m²/d will display similar patterns of water table fluctuation as another marsh characterized by highly porous soils and a transmissivity several orders of magnitude higher. Figures 10–12 suggest that this is especially true further away from the creek bank.

There also appears to be a seasonal influence on the sensitivity of the model to input parameter selection. This effect is discussed by comparing the standard error contours at the 36 m well locations along the three transects. The 36 m location is the first observation well outside of the creek bank zone, as it was defined previously. The data in Figure 17 indicate that at the 36 m well locations, the parameter region included within the 3 cm standard error contour is largest for transect 4, smallest for transect 3, and an intermediate value for transect 2. The transect 4 observations were made during the winter; the transect 3 observations were made during a very hot summer; and the transect 3 observations were made during average, spring-like conditions. During the winter, PET rates are lowest, and marsh water table dynamics are simplest, and hence reasonable predictions are obtained for transect 4 observations over the widest range of values. Conversely, during the summer, PET rates are at their highest annual values. As a result, there is more movement of water through the tidal marsh substrate, and accurate modeling of water table dynamics in the marsh interior along transect 3 requires more precise specification of transmissivity and specific yield values. During the spring, there are intermediate PET rates, and the model has intermediate sensitivity to parameter selection.

The sensitivity analysis above is a good example of the phenomena of equifinality [Beven and Binley, 1992; Beven, 1993; Freer and Beven, 1996; Brazier et al., 2000], but its conclusion that similar water table predictions are obtained over a wide range of transmissivity and specific yield values seems to contradict the recent findings of other researchers. Using a numerical model, Ursino et al. [2004] and Silvestri and Marani [2004] found that the substrate hydraulic conductivity is an important determinant of soil saturation levels in a hypothetical tidal marsh. Specifically, these researchers suggest that at a relatively low saturated hydraulic conductivity, a persistent unsaturated zone is present below the soil surface, even after inundation of the tidal surface. Although the transmissivity of the hypothetical site and of Piermont Marsh are of the same order of magnitude, the hypothetical marsh is much narrower than Piermont. The total width of the hypothetical marsh was 10 m, while at Piermont transects almost 40–120 times wider. Figures 10–15 demonstrate a lower sensitivity of the present model to substrate parameters beyond the creek bank zone. However, because it is narrow, the hypothetical marsh would appear to behave entirely as a “creek bank.” This is significant since wider marshes have greater pore water “reserves” to supplement water that has left the marsh as lateral flow the creek bank walls.
Other differences between the two modeling approaches that appear to be relevant in explaining the contradictory results are (1) the tidal boundary condition considered: regular, sinusoidal, and monochromatic leading to regular surface inundation in the case of the hypothetical simulation, yet irregular at Piermont and (2) a shallower overall water table at Piermont owing to the presence of a seepage face. No seepage face was simulated in the hypothetical studies, as observed by Wilson and Gardner [2005]. It would be interesting to see if Ursino, Silvestri and coworkers would obtain similar results to those presented here if their numerical model was used to simulate Piermont Marsh conditions.

This said, Figures 7–9 and 16, as well as Table 1, also reveal some limitations to the present model's predictive abilities. One of the obvious weaknesses is that some input parameter pairs lead to better predictions closer to the creek, and some further back. This result is a relic of the assumption of uniform soil properties and a uniform marsh surface elevation across the transect in the model. The model's inability to reflect soil and topographic heterogeneity is likely one explanation for the poor predictions at the 6 m and 12 m locations along transect 3, between 27 May and 6 June. Similarly poor predictions were found at the 6 m, 12 m, and 18 m locations, between 28 April and 1 May along transect 2. During both of these instances, the boundary condition water table was not as high as the elevation assumed for the average marsh surface. Yet the observations suggest that an inundation occurred. It is possible that the marsh surface was actually slightly lower at these locations and local inundations did, in fact, occur in the creek bank region, raising the water table there and there alone. Another possible explanation is that some unknown flow process or spatially variable soil properties facilitated conveyance of water from noninundating high tides into the marsh at these locations. In either case, the model presented is not capable of simulating these phenomena.

Also not well modeled is the series of sharp water table drawdown observed in the immediate postinundation water table profile for all wells along transect 3, between 10 and 17 May. Interestingly, the same phenomena are well

![Figure 15. Results of the mean cumulative error sensitivity analysis for transect 4.](image)

| Table 3. Transmissivity and Specific Yield Pairs Used in Graphical Presentation |
|-------------------------------|-------------------|--------|
| Transect | Parameter Set | Specific Yield | Transmissivity, m²/d |
| 2 | 1 | 0.30 | 9.0 |
| 2 | 2 | 0.70 | 30 |
| 2 | 3 | 0.90 | 70 |
| 3 | 4 | 0.42 | 0.7 |
| 3 | 5 | 0.60 | 0.007 |
| 3 | 6 | 0.94 | 20 |
| 4 | 7 | 0.20 | 0.5 |
| 4 | 8 | 0.60 | 4.0 |
| 4 | 9 | 0.98 | 8.0 |
modeled along transect 2 (see, for example, the 6 m predictions between 14 and 19 May). Multiple attempts to improve the transect 3 predictions, using a wide range of specific yield and transmissivity pairs were unsuccessful.

Figure 16. Overlap in error contours along each transect. (a) Standard error, (b) cumulative error, (c) mean deviation, and (d) combined error contours. In most cases the 3 cm contours at each well location were used. Notes give details on the contour overlaps shown. In the combined error contour diagrams the standard error parameter spaces have been colored green, and the mean deviation parameter spaces have been colored blue. The numbers in Figure 16d refer to the parameter sets in Table 3. MR is the parameter set for the “model run.”

[70] Other imprecise predictions relate to irregular fluctuations in the well water table observations. For example, the model is not capable of predicting small amplitude oscillations of the creek bank water table found in some wells (for example, the 6 m predictions along transect 4...
between 22 and 27 December), or the infrequent daily drawdown of water table detected in the 24 m water table on transect 2 between 24 and 27 April. The latter appears to have occurred during daytime low tide events.

To test whether the model’s ability to predict poorly simulated localized phenomena such as water table fluctuations due to noninundating high tides or small-amplitude, high-frequency well water table fluctuations could be improved, model predictions were made for transect 3 using a specific yield of 0.6 and an unrealistically high transmissivity ($T = 100 \text{ m}^2/\text{d}$). While this input parameter set did, in fact, allow the predicted water table to better simulate these fluctuations, the greater transmissivity led to greater overall horizontal drainage, which resulted in predicted water tables across the transect that were approximately 15–20 cm lower, on average, than the observations. Thus higher transmissivity values lead to better creek bank predictions and lower transmissivity values lead to better predictions inland. Although we did not model it, at a reasonable transmissivity value, we would expect that a low specific yield in the creek bank would be more appropriate for creek bank predictions, while a higher specific yield would be more appropriate for interior predictions. These effects cannot be well simulated with the model presented, again because of its requirement of uniform soil properties.

We also tested whether errors in model structure might be significantly skewing the results obtained. Specifically, we used the model to predict the boundary conditions at the creek banks and the marsh midpoint. At the interior of the marsh, an over-height error is introduced as a result of the neglecting nonlinear effects in the periodic solution of Flow System 3 [Parlange et al., 1984; Li and Jiao, 2003]. However, for superposition to work, linearization of the equations is a requirement. The error associated with linearization was easily calculated by the procedure outlined by Parlange et al. [1984] and later by Li and Jiao [2003]. With an amplitude of 30 cm estimated for the levee well boundary condition, and an aquifer depth of 2.7 m, the error introduced by linearization is, at most, approximately 1 cm at $x = L/2$, and nearly insignificant given the observation error.

The second potential error associated with model structure errors is at the creek bank, and specifically the boundary condition embodied by equation (19) (i.e., $d_3(x, 0) = 0$). As pointed out previously, $d_3(x, 0)$ as expressed by equation (20) does not equal zero. To quantify this error, we simulated $d_3$ at $x = 0$ for transect 3, and plotted the entire set of predictions with the observations in Figure 18, where this error would be the greatest. As shown in Figure 18 and its two insets, the model adequately replicates the boundary condition, suggesting that the error associated with this model structure inaccuracy is negligibly small, and even so, diminishes rapidly with time.

A final potential source of error in model structure is the ad hoc approximation used to simulate the fast drainage during noninundating high tides by preferential flow through macropores in the creek bank walls. To assess the error associated with this assumption, we recalculate the transect 3 predictions at $x = 6 \text{ m}$ without the preferential flow approximation. We picked this location along this transect because as shown in Table 2, this was the simula-
tion that invoked the preferential flow approximation to the greatest extent. This exercise produced negligible differences in the errors computed, likely because of its infrequent implementation in the simulation (see Table 2).

6. Conclusions

The mathematical model developed is an approximation of the major mechanisms of water flow observed at Piermont Marsh. As an irregularly flooded wetland in the temperate zone, its hydrology may differ from tidal wetlands in other places, with different hydroperiods. Nonetheless, for this site, the results of the model sensitivity analysis are informative in both assessing the limits of this model’s validity, and in explaining a number of aspects of the observed tidal marsh hydrology observations presented by Montalto et al. [2006].

Within a wide range of specific yield and transmissivity values, the model developed can generally predict the temporal and spatial fluctuations of the water table over much of Piermont Marsh to within the error threshold of the initial observations. The range of parameter values corresponding to reasonable standard errors appears to increase with distance from the creek bank. This is explained by the fact that subsurface flow dynamics are more complex closer to the creek. Seasonally, it appears that a more restricted set of transmissivity and specific yield parameter pairs is required for acceptable predictions in the summer, than in the winter, likely as a result of the increased importance of evapotranspirative fluxes during the warmer weather.

Although the model, as presented, can be used to compare, in general terms, the implications of broad differences in transmissivity, specific yield, marsh width, and marsh surface elevation on the water table fluctuations in tidal wetlands like Piermont Marsh, its use in more detailed ecohydrologic research will require additional refinement, especially in cases where small differences in the position of the water table may have important impacts on, for example, oxygen availability and wetland biogeochemistry. An example where this might be the case involves vegetation zonation, which is itself likely a function of multiple factors including soil saturation, but also hydroperiod, and salinity [Silvestri et al., 2005]. One way of accomplishing this goal might involve comparing our results with numerical modeling efforts that can simulate the spatial heterogeneity of topographic and edaphic properties at Piermont Marsh.

Model validation revealed a nondeterministic quality of the linearized, one-dimensional Boussinesq equation as solved. The equifinality result needs to be considered in any analysis attempting to use this model to make generalizations about hydrologic repercussions of a given set of marsh substrate characteristics. In a more general sense, the finding of equifinality here likely also carries implications for other applications with a (perched) groundwater table close to the surface and periodic rain events.

Notation

- \( d(x, t) \) elevation of the water table.
- \( d_1(x, t) \) solution 1: the gradual decay in water table elevation during periods of marsh surface exposure.
- \( d_2(x, t) \) solution 2: the net effect of precipitation and evapotranspiration on the water table elevation.
- \( d_3(x, t) \) step 1 of solution 2: the cumulative sum of all net meteorological fluxes through time, \( t \), assumed to act uniformly on the transect water table.
- \( d_4(x, t) \) step 2 of solution 2: the position of the water table as a result of all net meteorological fluxes through the marsh surface and all associated movement of water through the creek bank occurring during a given day.
- \( d_5(x, t) \) solution 3: tidally induced oscillations of the marsh water table.
- \( d_{\text{avg}} \) the average elevation of the levee boundary condition for the interval between inundation events.
- \( D \) constant aquifer thickness.
- \( h_{\text{ej}} \) the total daily precipitation minus the total daily evapotranspiration on day \( j \).
- \( K_s \) saturated hydraulic conductivity (assumed uniform, constant, isotropic).
- \( L \) distance between the two parallel tide creeks.
- \( P(t) \) precipitation.
- \( \text{PET}(t) \) potential evapotranspiration.
- \( T \) aquifer transmissivity.
- \( x \) time.
- \( w(t) \) net accretion into the control volume, \( P(t) - \text{PET}(t) \).
- \( \alpha_i \), \( \omega_i \), \( \alpha_i \) amplitude, angular velocity, and phase angle of the periodic boundary condition at the creek bank.
- \( x_i, b, m \) fitting parameters used to describe the initial condition water table profile.
- \( S_y \) specific yield.

References


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