

ANALYSIS OF RUNOFF-PLAT EXPERIMENTS WITH VARYING INFILTRATION-CAPACITY

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Experimental data--Research Bulletin 280, Missouri Agricultural Experiment Station, gives the results of runoff plat experiments by Jesse H. Neal [see 1 of "References" at end of paper]. The same soil was used in all the experiments. This was Putnam soil, taken from the surface five to six inches, from a timothy meadow. The soil was placed in a tank 12 feet long, 3.63 feet wide, and 26 inches deep. Water was applied by an overhead sprinkling applicator. Neal states: "Before each run the soil was dried and cultivated to a depth of 4 inches . . . After the moisture-content of the surface four inches of soil had been reduced to between two-thirds and three-fourths of the capillary capacity, the surface-soil was leveled off and worked down to approximately the proper height. . . After the soil was approximately leveled off with a rake, a wooden templet was drawn the length of the box and all excess soil scraped off. Since the templet left the soil in a smooth condition, the rake was run lightly back and forth across the templet to simulate the condition of a good seed-bed. . . The surface-inch of soil was further dried to between one-fourth and one-half the capillary capacity before applying the rain." The rain was applied at intensities of approximately 4 inches, 3 inches, 2 inches, 1.5 inches, and 0.9 inch per hour, and the experiments were carried out with surface-slopes of 0, 1, 2, 4, 8, 12, and 16 per cent.

Soil-moisture determinations were made to depths of zero to one inch and zero to four inches before each experiment. The runoff, including solid matter, was measured in a tank at 10-minute intervals and the weight of solid material in the runoff was determined [for further details of methodology, see Neal's paper].

These experiments supply a set of data with the same soil and, as nearly as possible, the same surface-condition in each experiment but with varying rain-intensities and varying slopes. Neal has given infiltration-capacities for different time-intervals, determined by subtracting the net or water runoff from the total water applied. It was found that the infiltration-capacity decreased with time of application of water. These determinations do not, however, take into account the effect of surface-detention and give too high initial infiltration-capacities.

Neal found no consistent relation between surface-slope and infiltration-capacity. He did find, however, a fairly consistent relation between initial soil-moisture content and infiltration-capacity during the first ten minutes of rain. He has discussed at some length the relation of soil-losses to various factors and found a consistent relation between total soil-loss in pounds per inch of water applied, and the two variables, slope and rain-intensity.

Neal's experiments supply excellent data for determining the relation of infiltration-capacity to duration of application of rain and also for a study of various hydrologic factors affecting surface-runoff and erosion under conditions where the soil and soil-surface remain sensibly constant. They also show the variation which may be expected in such factors as depression-storage and maximum and minimum infiltration-capacity on small sample areas due to unavoidable variations in soil-condition. Presumably substantially the same variations would be encountered in different parts of a larger area, other things equal. It is probably true that the average of the experiments for a given slope and rain-intensity represent nearly the average condition which would pertain on a larger area with the same soil, soil-surface condition, slope, and rain-intensity.

In a previous paper [2] the author has given methods of analysis of runoff-plate experiments in cases where the infiltration-capacity remains sensibly constant from the start. The principal purpose of the present paper is to extend these methods to cases where, as in Neal's experiments, the infiltration-capacity decreased with duration of application of rain, ultimately becoming sensibly constant. The present paper is therefore supplemental to Bulletin 280 and covers somewhat different lines of analysis of the same experiments. The conclusions in the two papers do not in any way conflict. The author wishes to point out that the present paper is possible only because of the high quality and accuracy of Neal's experiments, as some of the methods used herein are sensitive and are vitiated by relatively minor errors in the experimental data. Professor Neal has kindly supplied data of measured runoff for different time-intervals in each experiment, together with time of beginning and ending of surface-runoff. These data have been used as a basis of the analyses in this paper.

Determination of varying infiltration-capacities from runoff-graphs--Referring to Figure 1, let $t = 0$ at the beginning of rain, $i =$ the rain-intensity in inches per hour, q_s the surface-

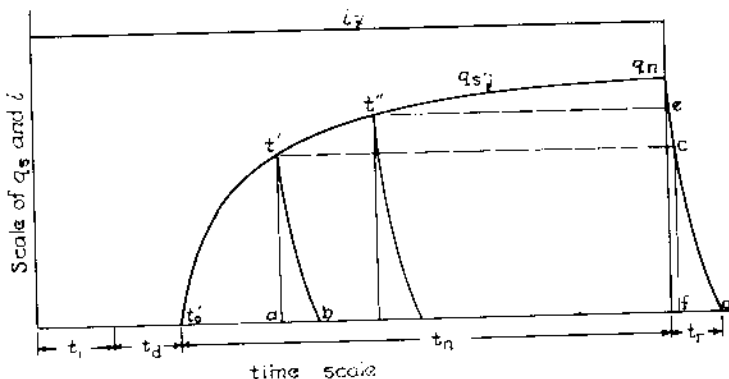


FIG. 1—DETERMINATION OF INFILTRATION-CAPACITY

and, in the interval t_r , runoff occurs from surface-detention only. It can be shown that for 75 per cent turbulent flow the amount of residual runoff is approximately

$$Q_r = (1/3) Q_n t_r \quad (1)$$

where Q_n is the runoff-rate at time t_n . The average fraction of the area covered with surface-detention during residual runoff (exclusive of depression-storage on parts of the area from which runoff has ceased) is $1/3$. The infiltration (with the same exception as to depression-storage) during residual runoff is the same as that from the entire area during a time interval $(t_r/3)$. Since the surface-detention V_d is sensibly the same at the beginning and end of runoff, the total infiltration in the interval $(t_n + t_r/3)$ is $(P - Q)$, where P is the rainfall for the interval t_n and $Q = (Q_n + Q_r)$. Q_n is the total runoff to the time t_n . Q_r is included since this runoff is derived from rain falling in the interval t_n . The average infiltration-capacity for the interval $(t_n + t_r/3)$ is

$$f_a = (P - Q)/(t_n + t_r/3) \quad (2)$$

Since t_r is usually small relative to t_n , no large error results from neglecting t_r in computing f_a .

Now suppose the rain had ended at some time t' preceding t_n . Then the average infiltration-capacity for the interval t'_0 to t' would again be

$$f_a = [P' - (Q' + Q'_r)]/[(t' - t'_0) + t'_r/3] \quad (3)$$

The residual runoff-graph would have the same form as before but would start at a lower point on the hydrograph, and the value Q'_r would be reduced by the area cd (Fig. 1). Making this correction in Q_r , the average infiltration-capacity from the beginning of runoff to the end of any given time-interval can readily be obtained.

Similarly, the average infiltration-capacity for any time increment t' to t'' between two sets of observations can be obtained by the equation

$$f = (\Delta P - \Delta Q)/(t'' - t' + (t''_r + t'_r)/3) \quad (4)$$

where $\Delta Q = (Q'' + Q''_r) - (Q' + Q'_r)$, the values of Q being given and those of Q_r determined in each case as above described. The values of t'_r and t''_r can be scaled from the residual runoff-graph, as, for example, $t_d = t'_r$ on Figure 1. Except for early stages of runoff, the values of $(t'_r/3)$ and $(t''_r/3)$ are nearly the same. Assuming them equal, this term disappears by subtraction. It is only in computations of f for earlier stages of surface-runoff that the $(t_r/3)$ term is required. This method of computation of the average value of f for different time-increments has the effect of reducing the surface-detention, including depression-storage, to a constant amount (V_d) at the beginning and ending of the time-increment, so that surface-detention need not be otherwise determined or taken into account.

A sample sheet of data and computations is given on Table 1. Column (2) gives the time-

TABLE 1 - ANALYSIS OF NEAL'S EXPERIMENT NO. 9

increments at the ends of which Q was observed. Columns (3), (4), and (5) give the increments of rainfall, the rain-intensity, and the mass-rainfall. Columns (6), (7), and (8), relating to soil-losses, are referred to later on. Columns (9), (10), and (11) give the runoff-increments, the runoff-intensity q_s at the end of each time-interval, and the mass-runoff, respectively. Column (12) gives the increment of rainfall for the given time-interval. This is the same as column (3) excepting for the first-increment. For the first time-increment the rainfall is computed for the part of the increment during which runoff occurred. In this case $t_0 = 3$ minutes. ΔR is computed for 7 minutes. Column (13) equals the total runoff to the end of the given time-increment plus the residual runoff which would have occurred if rain had ended at the end of the given increment, or at the time shown on the same line in column (3); this is designated Q_0 . Column (14) gives the ΔQ_0 , or the increment of runoff resulting from the rain which fell in the given time-interval. This is the difference between Q_0 on a given line and that on the preceding line. Column (15) is the increment of total infiltration, F, for the given time-increment, obtained by subtracting the item in column (14) from that in column (12). This, in turn, divided by the time-increment, gives the average infiltration-capacity or the infiltration-capacity f at the mid-point of the given time-increment, as shown in column (16). Computation of the curve of f may be checked by computing f_0 for the entire runoff-interval by equation (2), then averaging the ordinates of the plotted curve for the same time. The two quantities should be sensibly equal.

Relation of infiltration-capacity to rainfall-duration.-As first noted by Horton [2] and subsequently by Neal [1], there is in general a decrease of infiltration-capacity during the early stages of a storm. Exceptions will occur: (1) where the soil is already at its minimum infiltration-capacity; (2) in some regions, particularly on the western Great Plains, a moderate increase of infiltration-capacity has been noted [3] in the earlier stages of application of water to sprinkled plots. The increase occurred on areas subject to dust-storms and may have been due to washing away of fine dust deposited on the soil-surface and into the entrances to soil-pores.

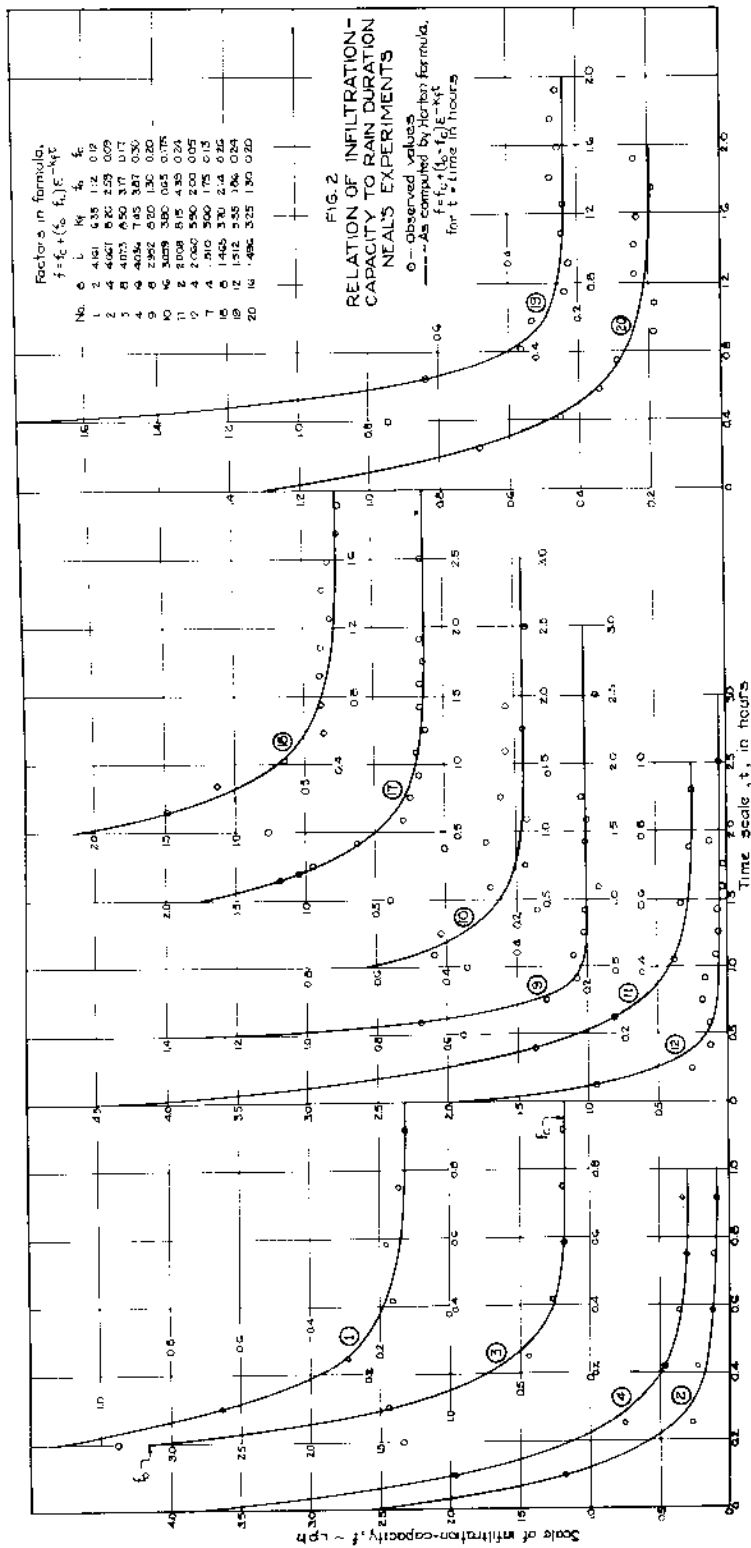
There are at least three reasons why infiltration-capacity may decrease in the earlier stages of rain:

- (1) Swelling of colloids and closing of soil-cracks and sun-checks.
- (2) Inwashing of fine material to the surface-pores in the soil. Where surface-

Slope %	Time Minutes	Rainfall		Soil loss		Runoff										Computation of f											
		ΔR in. (3)	i in. h. (4)	Δ lb. (5)	Σ lb. (6)	Δ in. (7)	q_s in. h. (8)	Q in. (9)	Q_0 in. (10)	ΔP in. (11)	ΔQ_0 in. (12)	F in. (13)	f in. h. (14)	Comp'd f in. h. (15)	ΔF in. (16)	ΔQ_0 in. (17)	Q_0 in. (18)	Q+F in. (19)	δ_5 in. (20)	δ_5 in. (21)	Q_0 in. (22)	δ in. (23)	δw Sq. Ft. (24)	V (25)			
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
0	10	.492	2.952	.482	4.82	2.125	2.48	.344	3.44	.254	2.54	.318	3.18	.100	1.00	.716	3.24	.324	.072	.072	.0026	.163	.0469	.053			
0	20			3.64	15.48	9.39	4.31	2.562	4.92	6.93	4.39	3.18	3.18	.100	1.00	.716	3.24	.324	.072	.072	.0026	.163	.0469	.053			
0	30			1.476	4.00	0.480	13.39	4.5	2.700	11.29	4.53	2.34	2.34	.140	1.40	1.269	3.24	.324	.072	.072	.0026	.166	.0498	.054			
0	40			1.968	3.34	0.401	16.73	4.57	2.700	15.60	4.52	2.40	2.40	.175	1.75	1.755	3.24	.324	.072	.072	.0026	.175	.0525	.051			
0	50			2.460	3.40	0.408	20.13	4.57	2.922	2.667	4.92	0	0	.20	2.00	2.275	3.24	.324	.072	.072	.0026	.184	.0642	.072			
0	60			2.952	2.90	0.348	23.03	4.64	2.784	2.531	2.549	2.04	2.04	.241	2.41	2.772	3.24	.324	.072	.072	.0026	.196	.0718	.074			
0	70			3.444	2.87	0.344	25.90	4.65	2.790	2.996	3.013	1.62	1.62	.20	2.0	3.270	3.24	.324	.072	.072	.0026	.208	.0731	.080			
0	80			3.936	2.37	0.284	28.27	4.32	2.992	3.428	3.412	3.78	3.78	.20	2.0	3.735	3.24	.324	.072	.072	.0026	.217	.0741	.085			
0	90			4.428	2.25	0.270	30.52	4.18	2.508	3.646	3.625	4.16	4.16	.20	2.0	4.166	3.24	.324	.072	.072	.0026	.229	.0757	.098			
0	100			4.920	2.14	0.259	32.76	4.83	2.718	4.293	4.317	4.53	4.53	.20	2.0	4.572	3.24	.324	.072	.072	.0026	.243	.0784	.130			
0	110			5.412	2.40	0.288	35.00	4.59	2.754	4.758	4.714	4.57	4.57	.20	2.0	5.064	3.24	.324	.072	.072	.0026	.259	.0824	.160			
0	120			5.904	2.61	0.313	38.27	4.43	2.688	5.201	5.215	3.06	3.06	.20	2.0	5.640	3.24	.324	.072	.072	.0026	.278	.0864	.190			
0	130			6.396	4.55	0.251	42.02	4.57	2.760	7.981	7.952	1.69	1.69	.20	2.0	6.220	3.24	.324	.072	.072	.0026	.300	.0920	.210			
0	140																										
0	150																										

(a) Runoff commenced at 3 minutes

Only total application for this expt given



erosion occurs, water entering the soil-pores is charged with fine material in suspension, tending to clog the surface-pores. Even though the surface-soil is itself carried away, the clogging process still continues.

(3) Rain-packing--Especially in the earlier stages of intense rains, direct impact of rain-drops on the soil compacts the soil-surface and decreases the infiltration-capacity. Except on flat areas, the soil-surface is not as a rule wholly covered except with a thin film of water during rain, and on the portions not more deeply covered, rain-packing continues until the soil-surface reaches a maximum density.

The first and third conditions partake of the nature of exhaustion-phenomena and apparently follow the law of diminishing returns or the inverse exponential law in some form.

The infiltration-capacities derived from typical graphs of Neal's data are shown by circles on Figure 2. It was impossible to cover the interval from $t = 0$ to t_0^1 , the beginning of runoff. However, the platted points apparently belong to curves which start at some finite value f_0 for $t = 0$. The infiltration-capacity then decreases as the duration of application of water continues, the value of f approaching a constant minimum value f_c asymptotically. These characteristics are common to exhaustion-phenomena and suggest that the curves corresponding to the platted points can be represented by a general equation of relation of infiltration-capacity to duration of rainfall of the form

$$f = f_c + (f_0 - f_c)e^{-K_f t} \quad (5)$$

where f = infiltration-capacity, inches per hour, at time t , in hours; f_c = initial infiltration-capacity at time $t = 0$; f_0 = minimum constant infiltration-capacity; K_f is constant for a given curve. (This equation has been found to fit accurately the results of ring-infiltration experiments by Free, Browning, and Musgrave, and sprinkled-plot experiments on natural desert soils at Tucson, Arizona, by the United States Soil Conservation Service.)

The experiments give f_c directly. To determine the other two constants, f_0 and K_f , the following method was used. Smooth curves were drawn to represent the platted points within the range for which they were determined. From each curve two pairs of values, $t_1 f_1$ and $t_2 f_2$, were taken off. Then from equation (5)

$$f_1 = f_c + (f_0 - f_c)e^{-K_f t_1} \quad (6)$$

$$f_2 = f_c + (f_0 - f_c)e^{-K_f t_2} \quad (7)$$

from which, by transposition

$$f_0 - f_c = (f_1 - f_c)e^{K_f t_1} = (f_2 - f_c)e^{K_f t_2} \quad (8)$$

The only unknown quantity in the two right-hand terms is K_f . The values of the second and third terms of equation (8) were computed for each of a series of assumed values of K_f and the resulting numerical values of the second and third terms of equation (8) were platted in terms of K_f , as shown on Figure 3. The point of intersection of the smooth curves drawn through the two series of platted points gives at once the value of K_f and also $(f_0 - f_c)$. Since f_c is known, f_0 can then be determined.

On Figure 2 the solid lines were computed from equation (5), with the constants appropriate to each curve. Aside from obvious discordancies in the data in some cases, due to lack of details of variation in rain-intensity, there is good agreement between the platted points and the computed curves.

While equation (5) may not actually represent the law governing the physical processes involved, this equation is rational in form, since it not only represents the observed data within the range of observation but also gives results in agreement with known facts for the limiting or boundary conditions.

The values of K_f , f_0 and f_c , determined from the experimental data in conjunction with equation (5), are given in columns (4), (5), and (10) of Table 2.

Time required to attain constant infiltration-capacity--It will be noted that in all these experiments the infiltration-capacity becomes sensibly constant in the time ranging from one-half

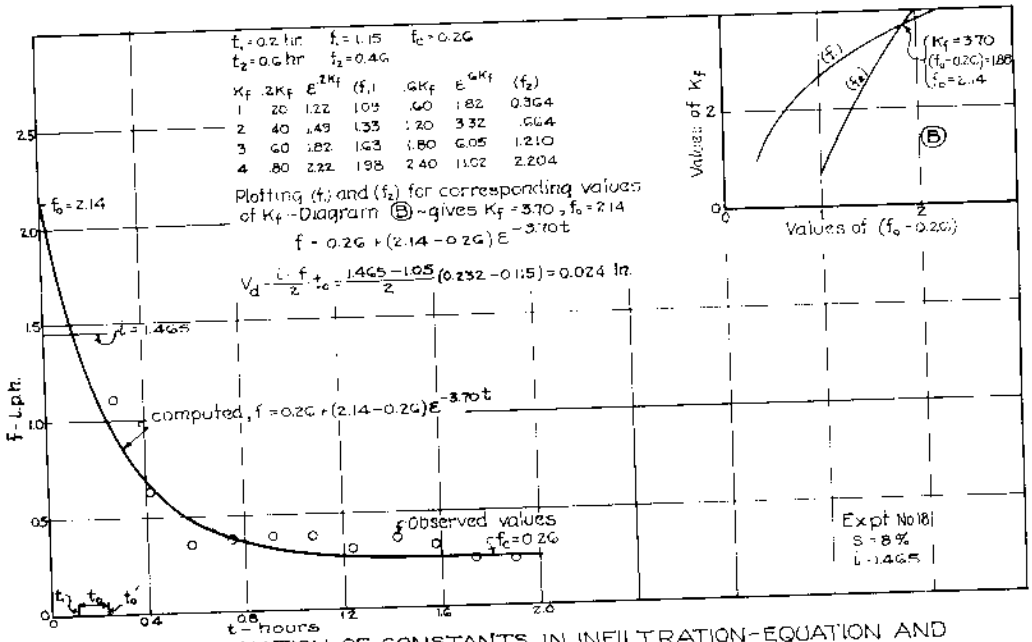


FIG. 3 - DETERMINATION OF CONSTANTS IN INFILTRATION-EQUATION AND COMPUTATION OF V_d

TABLE 2 - INFILTRATION-CONSTANTS - NEAL'S EXPTS.

No.	L	S	f_0	f_c	V_d	t_0	OC	q_{60}/OC	K_f	m_0	f_0'
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
1	4.161	2	1.41	0.12	0.147	0.050	4.041	0.992	6.35	11.8	0.72
3	4.073	8	3.47	0.17	0.48	0.12	3.903	1.002	8.10	13.5	2.62
2	4.067	4	2.87	0.09	0.67	0.42	3.977	1.006	8.20	12.8	1.97
4	4.036	16	3.87	0.30	0.26	0.42	3.736	0.961	7.45	10.5	1.88
	4.084		2.89	0.17	0.672	0.44		0.990	7.62		1.80
8	3.101	4	2.29	0.09	0.068	0.050	3.031	0.980	12.20	19.9	2.04
10	3.059	16	0.63	.17	.123	.050	2.889	1.002	3.80	20.7	0.71
6	3.030	1	3.27	.16	.030	.050	2.870	.993	7.10	19.5	2.65
7	2.993	2	3.03	.20	.040	.083	2.793	.992	5.65	12.1	2.08
9	2.952	8	1.63	.20	.080	.050	2.752	1.011	8.20	19.7	1.44
	3.027		2.17	0.16	0.068	.057		.996	7.39		1.78
12	2.000	4	2.25	0.05	0.021	0.058	2.010	0.971	5.90	19.8	2.21
14	2.026	16	2.53	.28	.019	.067	1.746	.973	6.35	12.6	1.87
13	2.017	8	4.40	.25	.001	.021	1.767	1.008	6.80	10.9	1.97
11	2.008	2	4.39	.24	.014	.050	1.768	.991	8.15	11.5	2.70
	2.028		3.39	0.20	0.14	0.49		.986	6.80		2.19
19	1.512	12	2.49	0.24	0.010	0.050	1.272	0.974	3.55	13.4	1.52
17	1.510	4	1.75	.13	.011	.075	1.380	.972	3.00	16.4	1.77
16	.503	2	1.48	.16	.078	.217	1.343	.936	2.75	18.6	1.42
20	1.496	16	1.53	.26	.040	.150	1.296	.953	3.25	13.6	1.69
15	1.484	1	1.57	.18	.040	.158	1.304	.919	3.20	17.4	1.66
18	1.465	8	2.14	.26	.023	.117	1.205	.954	3.70	11.6	1.33
	1.495		1.81	0.19	0.034	.128		.951	3.57		1.46
22	0.900	8	2.28	.23	.064	.117	0.670	.956	4.60	24.0	2.56

hour to two hours. This time-interval is of great importance in relation to land-use and flood-control. For practical purposes f may be considered sensibly constant when it attains a value within one per cent of its final constant value. The time t_c required can be determined as follows: Let $f = 1.01 f_c$. Then from equation (5)

$$f = 1.01 f_c = f_c + (f_0 - f_c)e^{-K_f t_c}$$

or

$$f_c = 100 (f_0 - f_c)e^{-K_f t_c}$$

and

$$t_c = (1/K_f) \ln 100(f_0 - f_c)/f_c \tag{9}$$

Total or mass-infiltration--Given the constants f_0 , f_c , and K_f , equation (5) provides a ready means of determining the total or mass-infiltration for any time-interval. Since

$$F = \int_0^t (f_c + (f_0 - f_c)e^{-K_f t}) dt \tag{10}$$

or

$$F = f_c t - (1/K_f) (f_0 - f_c)e^{-K_f t} + \text{constant}$$

when $t = 0$, $F = 0$; therefore, constant = $(1/K_f) (f_0 - f_c)$

and

$$F = f_c t + (1/K_f) (f_0 - f_c) (1 - e^{-K_f t}) \tag{11}$$

If there is a period t_1 of antecedent rainfall during which $i < f$, the computation of mass-infiltration by equation (11) must begin at the point where f and i become equal. To obtain the total infiltration from the beginning of rain there must be added to this the quantity it_1 , or the infiltration for the period of antecedent rainfall, before depression-storage begins to fill.

Depression-storage--One of the facts brought out by these studies is that, with the exception of rain-intensities of three and four inches, the initial infiltration-capacity f_0 was usually greater than the rain-intensity i , so that there was a period of antecedent rainfall t_1 during which the soil absorbed the rain, and neither runoff nor depression-storage occurred. The time t_1 is that at which infiltration-capacity f and rain-intensity i become equal and can be taken directly from the f -curves. Using t_d to represent the time required to fill depression-storage and t_0' the time at which surface-runoff began, then when depression-storage begins to fill, $f = i$. when it is filled and runoff begins, or at the time t_0' , f has some value f' . The mean infiltration-capacity while depression-storage is filling is, approximately

$$(i + f')/2$$

The volume of depression-storage can therefore be computed in all cases by the equation

$$V_d = [i - (i + f')/2] t_d = (i - f'/2) t_d \tag{12}$$

Column (6) of Table 2 contains the computed values of V_d . In general it is to be expected that depression-storage will decrease as slope increases, other things equal. The average values of the depression-storage for different slopes shown by these experiments are as follows:

Slope in per cent	Number of experiments	V_d in inches
0	
1	2	0.047
2	4	0.053
4	4	0.043
8	5	0.022
12	1	0.033
16	4	0.055

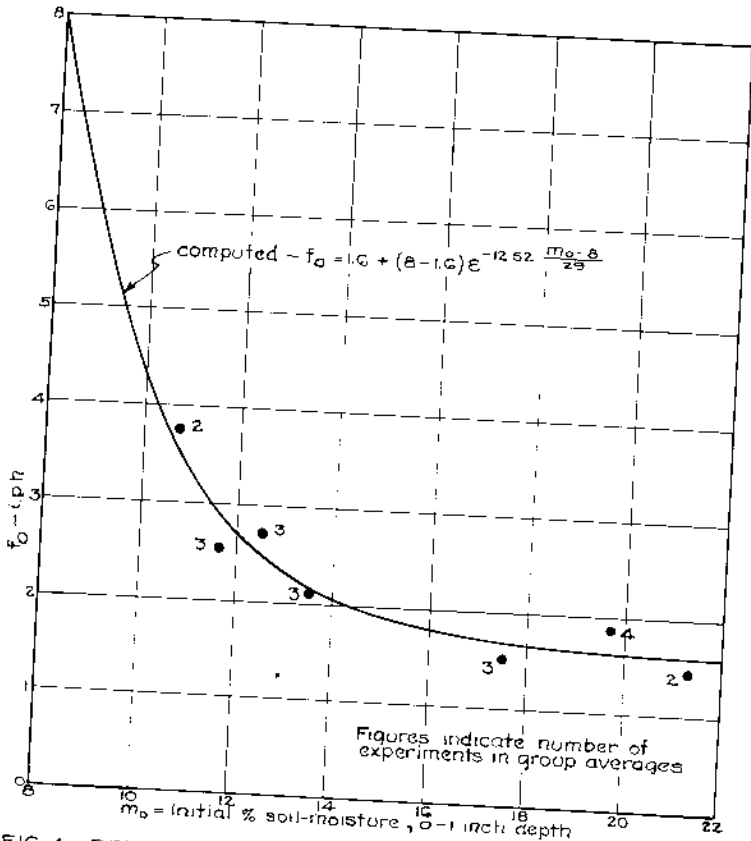


FIG. 4 - RELATION OF f_0 TO INITIAL SOIL-MOISTURE CONTENT AT 0-1 INCH DEPTH

For slopes of 2 to 12 per cent, the experiments show marked evidence of decrease of depression-storage with increasing slope. The high value of V_d for slopes of 16 per cent is largely the effect of one abnormally high value of V_d . As a result the variation of depression-storage with slope is partly masked by residual errors.

Relation of infiltration-capacity to soil-moisture--Neal found a close correlation between his computed average infiltration-capacity for the first ten minutes of application of rain to the initial soil-moisture. The validity of his results are, however, affected by the fact that depression-storage and antecedent rainfall in which $f > 1$ were not taken into account. A better basis of comparison of infiltration-capacity with initial soil-moisture seems to be possible using f_0 as a compare.

The initial soil-moisture to a depth of one inch, expressed in percentage of dry weight of soil, as given by Neal, was compared with the corresponding values of f_0 , taking group-means of experiments with closely similar initial soil-moisture. The results are shown on Figure 4. The curve is closely similar to that obtained by Neal.

A similar plotting of f_0 in terms of initial soil-moisture for a depth of one to four inches showed no definite evidence of correlation. This clearly indicates that the initial infiltration-capacity, at least, is primarily a function of the condition of the soil-surface.

The data on Figure 4 are well represented by the equation

$$f_0 = f_w + (f_d - f_w)e^{-K_m \frac{m_0 - m_d}{m_w}} \quad (13)$$

where m_w is the moisture-content, per cent dry weight of the soil in its initial surface-condition but fully wetted (to field-moisture-capacity), f_w is the infiltration-capacity of the soil in its

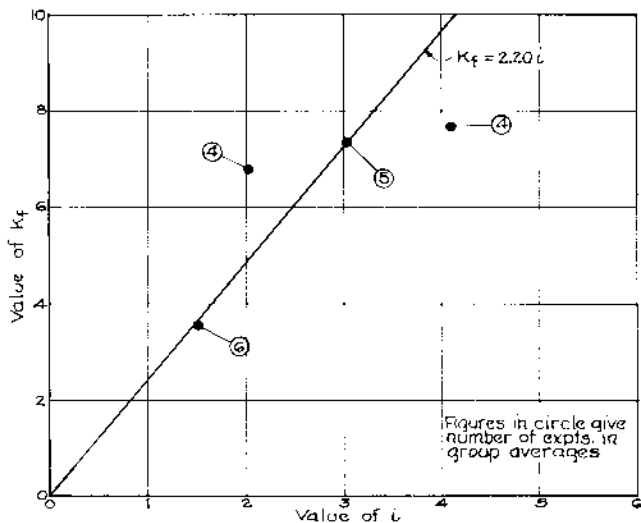


FIG. 5 - RELATION OF K_f TO RAIN-INTENSITY, UNCORRECTED DATA, NEAL'S EXPTS. IN EQUATION, $f = f_c + (f_0 - f_c)e^{-K_f t}$

initial surface-condition but wetted at least to field-moisture-capacity. m_d and f_d are the moisture-content and infiltration-capacity, respectively, of soil in its initial condition when air-dry. m_0 and f_0 are, respectively, the actual initial moisture-content and initial infiltration-capacity. m_w and m_d correspond approximately to moisture-content at field-moisture-capacity and the hygroscopic moisture-content, respectively. Neal's experiments give $f_d = 8$ i.p.n., $f_w = 1.6$ i.p.n., $K_M = 12.52$. The difference, $(f_d - f_w) = 6.4$ i.p.n., is the maximum effect of drying out of the soil-surface for this soil and method of treatment.

Since the soil was the same and the surface-treatment was the same in different experiments, the observed variations in f_0 shown in column (4) of Table 2 are apparently due to (1) variation in soil-moisture, (2) uncontrolled differences in surface-compactness, (3) variation in rain-intensity. The effect of variation

of initial soil-moisture can be largely eliminated by taking off from the curve on Figure 4 the values of f_0 for an assumed constant initial soil-moisture m_0' . For this purpose $m_0' = 16$ per cent was used. For 16 per cent soil-moisture the curve gives $f_0 = 1.80$ inches per hour. The observed values of f_0 were reduced to the basis of 16 per cent soil-moisture by assuming that the observed f_0 would have been changed by changing the initial soil-moisture content to 16 per cent in the same ratio that the corresponding values of f_0 on the curve are changed, or

$$f_0'/f_0 = f_a/f_b$$

where f_0' is the initial infiltration-capacity reduced to the basis of 16 per cent soil moisture, f_a is the value of f_0 given by the curve for the actual moisture-content, and $f_b = 1.80$ inches per hour for 16 per cent moisture. Consequently

$$f_0' = 1.8 (f_0/f_a) \tag{14}$$

The values of f_0' or the initial infiltration-capacities for 16 per cent moisture are shown in column (12) of Table 2. Correction to 16 per cent moisture reduces the range and variability of f_0 . The average of the observed values of f_0 is 2.24; the average of the values of f_0' is 1.81; the average of the soil-moisture, zero to one inch, was 15.4 per cent.

The preceding discussion lays a foundation for the prediction of infiltration-capacity on a soil of the type and with the method of cultivation used in Neal's experiments. For a large area, with the same soil and cultivation, it may be assumed that the average of f_c will apply in equation (5). From Table 2 this is 0.186; hence

$$f = 0.186 + (f_0 - 0.186)e^{-K_f t} \tag{15}$$

The constant K_f is an exponential parameter which measures the time required for the infiltration-capacity to drop from its initial to its minimum value as the result of rain-packing and inwashing, particularly the former. It is presumable that the rate of rain-packing, and the consequent reduction of infiltration-capacity thereby, increases with rain-intensity, or the higher the rain-intensity, the shorter the time t_c required to reduce the infiltration-capacity from that corresponding to initial soil-moisture, to the minimum value.

The larger the value of K_f the shorter the time t_c . Hence K_f should increase as the rain-intensity i increases. Neal's experiments afford an opportunity to test this hypothesis. The computed values of K_f were averaged by groups for the different rain-intensities and the results have been plotted on Figure 5. While not highly consistent, the data are well represented by the equation

$$K_f = 2.20 i$$

(15)

This expression is rational in form, since it makes f constant and equal to f_0 for zero rain-intensity and, on the other hand, the time required for the infiltration-capacity to reach its minimum value f_0 approaches zero as a limit as the rain-intensity i increases.

If the initial surface soil-moisture percentage is known, then equations (12), (15), and (16) provide a means of predicting the march of infiltration-capacity during a subsequent rain. For example, let $i = 3.0$ i.p.n. for a soil the same as used by Neal, with $m_0 = 8$ per cent, $m_w = 39$ per cent, and $f_w = 1.6$ i.p.n. Let it be required to determine the infiltration-capacity for an initial soil-moisture $m_0 = 12$ per cent. From equation (13) $f_0 = 2.70$ i.p.n. From equation (16) $K_f = 6.60$. Substituting these values of f_0 and K_f in equation (15) gives

$$f = 0.186 + (2.10 - 0.186)e^{-6.60t}$$

for time t in hours. Solution of this equation for different values of t will give the march of infiltration during the subsequent rain.

Relation of f_0 to antecedent rain--Ordinarily the initial surface soil-moisture m_0 is unknown. In order to make equations (13), (15), and (16) of the utmost practicable value it is desirable to have some simple correlation between time elapsed since the last preceding rain and the surface soil-moisture at the beginning of a given rain. Presuming that the antecedent rain penetrated the soil to a sufficient depth so that the rate of drying out would not be greatly affected by moisture-condition below the depth of penetration, then it appears that at the end of the last antecedent rain, the surface soil-moisture would have been equal to the field-moisture-capacity m_f . At a time t_0 subsequent thereto the surface soil-moisture will be reduced by an amount dependent on the evaporation-rate. It appears probable that a fairly good correlation between initial moisture-content and time elapsed since the last preceding rain could be worked out but thus far this has not been done. Quite probably it is an inverse exponential law of a form

$$m_0 = m_f + (m_f - m_n)e^{-K_m t} \quad (17)$$

where m_f = the field-moisture-capacity, m_n = the hygroscopic moisture-capacity, and m_0 = moisture remaining at time t , and K_m depends on the evaporation-rate.

It is possible that a simple linear equation

$$m_0 = m_f - (m_f - m_0) (t_0/t_d) \quad (18)$$

will meet practical requirements. In this equation t_0 is the time required for the soil-surface to become air-dry or to attain a condition of equilibrium with air of a given temperature and humidity. This is a function of the evaporation-rate but since there is a high degree of correlation between evaporation and air-temperature, and since temperature-data are much more commonly available than evaporation-data, it is possible that t_0 can be expressed with sufficient accuracy in terms of air-temperature alone.

Minimum infiltration-capacity--Normal minimum infiltration-capacity may be defined as the infiltration-capacity of a soil-surface free from sun-checks and biologic structures (perforations by earthworm, insect, and root) which has been wetted to field-moisture-capacity long enough to permit full swelling of colloids and adjustment of the soil-structure to a stable field-condition. The author has closely duplicated determinations of f in such soils in repeated experiments and hence concluded that a given soil has a definite normal minimum infiltration-capacity [4].

A given soil-surface may not fully meet these conditions during a rain because of under- or over-packing, puddling of the soil-surface and inwashing of fine material, or because of the presence of biologic structures. Some variation of f_0 in runoff-plot experiments is therefore to be expected.

The following tabulation shows the number of values of the constant or final infiltration-capacity lying between given limits. For most of the experiments f_0 lies between 0.15 and 0.20 and the variations from the average value $f_0 = 0.186$ may apparently be attributed to differences in compactness of the soil, as it is known that infiltration-capacity is sensitive to variation of soil-porosity.

Distribution of values of f_0	Number of values
< 0.10	5
> 0.10 < 0.20	8
> 0.20 < 0.30	5
> 0.30 < 0.40	3
> 0.40 < 0.50	4

Compared with f_0 , the variation of minimum infiltration capacity of a given soil is confined to a relatively narrow range. Also it is to be noted that the minimum initial infiltration capacity for loose soil, unconsolidated by rain but fully wetted, as shown by Figure 4, is 1.0 inches per hour. The difference between this and 0.126 inch per hour, the average minimum f_0 after prolonged rain, may be attributed to rain packing or inwashing, or both.

Characteristics of hydrographs—The hydrographs or runoff-graphs for several of Neal's experiments are shown on Figures 6, 7, and 8. These hydrographs were drawn through all of the ob-

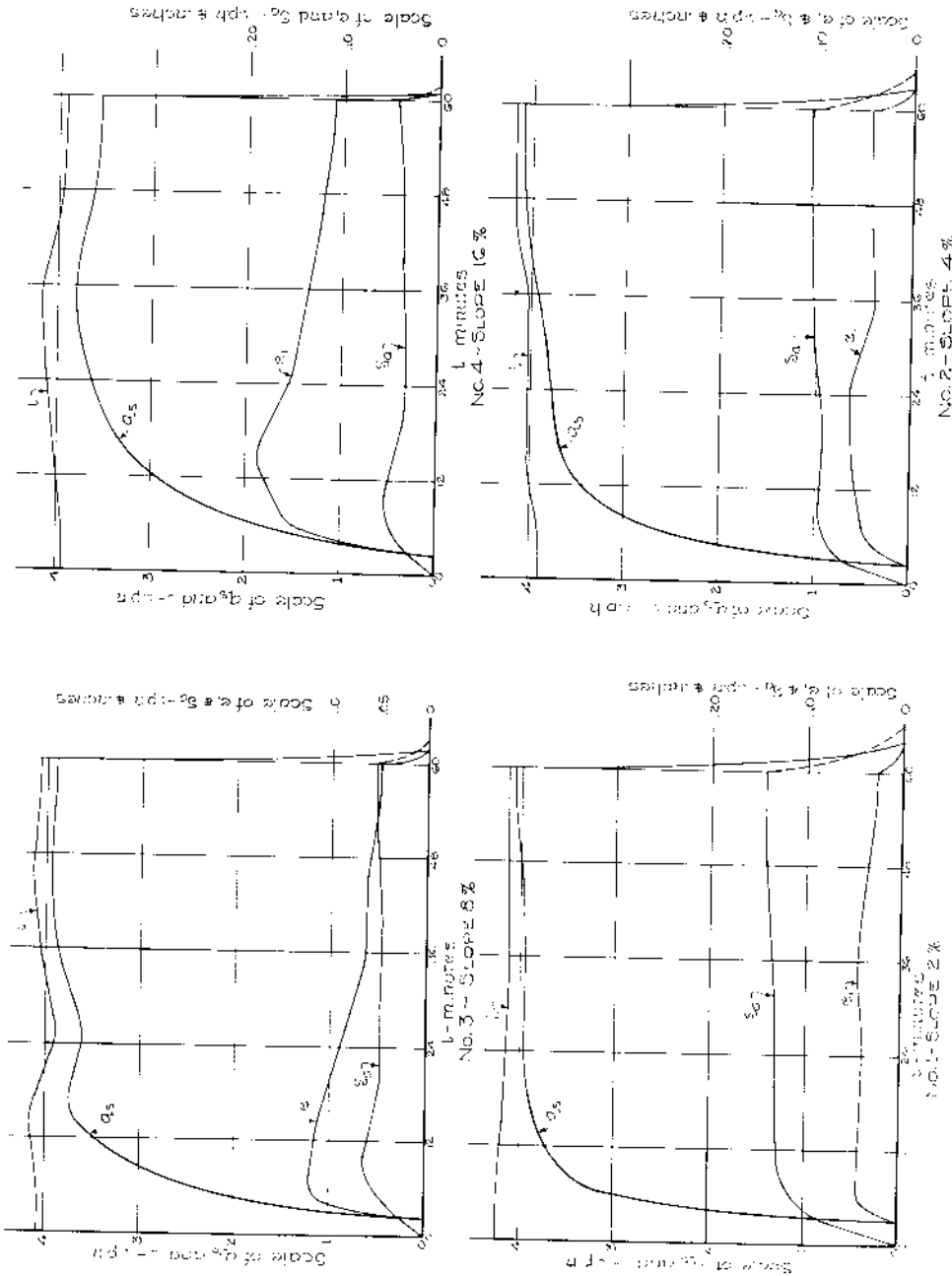


FIG. 6 ANALYSIS OF NEAL'S EXPERIMENTS, $i = 4.00$ lph

served points. They show in many instances distinct waviness, especially after the flow became stable. The presence or absence of waviness does not appear to be closely related either to the slope or rainfall-intensity. The fact that the wave-crests are usually 20 minutes or more apart, shows that these are not true rain wave-trains but that the waviness is due to alternate building up and breaking down of debris-dams on the soil-surface or to successive stages of initiation of gullies and consequent concentration of runoff [8]. Another characteristic of these hydrographs is the occurrence of relatively abrupt fronts in most instances. Because of this condition it

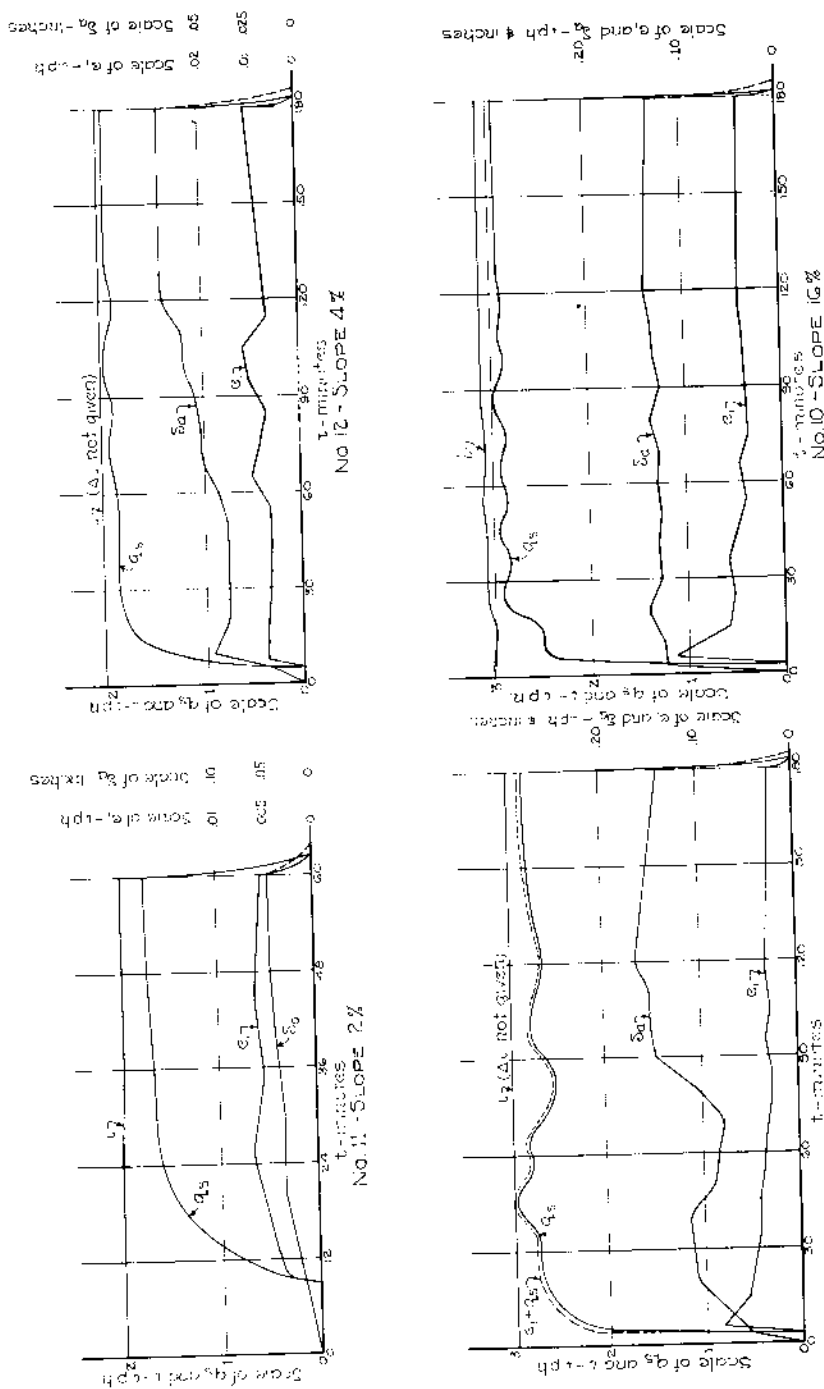


FIG 7 -- ANALYSIS OF NEALS' EXPERIMENTS, $v = 2$ AND 3 lph.

has been found impracticable except in a few cases to determine completely the law of overland-flow. Using the curves of infiltration-capacity for the same experiments (Fig. 2), the increments of surface-detention $\Delta\delta_A$ have been determined for successive time-increments $t_2 - t_1$ by the equation

$$\Delta\delta_A = t_1(t_2 - t_1) \cdot \frac{1}{t_1^2} \cdot (Q_2 - Q_1) + t_2(t_2 - t_1) \cdot \frac{1}{t_2^2} \cdot (Q_2 - Q_1) \quad (19)$$

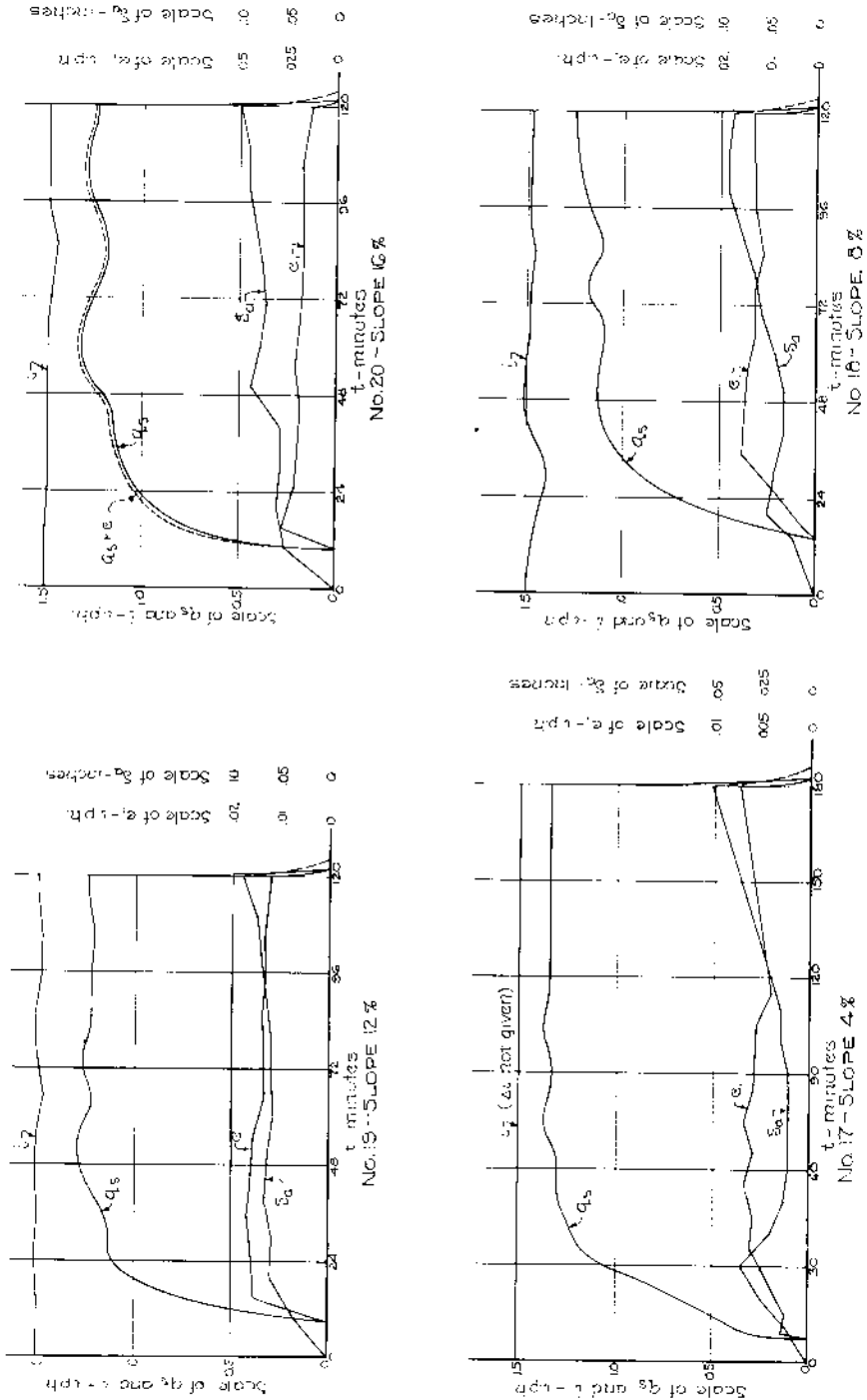


FIG. 8 - ANALYSIS OF NEAL'S EXPERIMENTS, $t = 15$ l.p.h.

where Q_2 and Q_1 are the total runoff at times t_2 and t_1 , from the observational data. f_a and f_d are the average rain-intensities and average infiltration-capacities for the given time-interval. By summation of values of $\Delta\delta_a$ the average detention-lines shown on Figures 6, 7, and 8 have been obtained. Neal has furnished the amounts of erosion in pounds on the 0.001-acre plot for each 10-minute time-interval. The erosion-rates for different time-increments were reduced to the basis of depth e_1 of the solid matter removed from the soil-surface in inches per hour. The specific gravity of the soil-material was 2.3, and the weight of a cubic foot of solid soil-material was 144 pounds. Hence for a 0.001-acre plot the rate e_1 , in inches per hour, is

$$e_1 = 12/(144 \times 43.56) \times (\text{pounds per hour}) = 0.00192 \times (\text{pounds per hour}) \quad (20)$$

Lines showing the erosion-rate e_1 have been added to Figures 6, 7, and 8.

In this connection it is suggested that the unit "inches depth per hour" of removal of solid matter is preferable to other units sometimes used in expressing erosion-rates from the ground-surface, since it does not involve the volume-weight of the soil, which varies for different soil-conditions, and thus enables experiments on different soils or on the same soil under different conditions to be directly compared. Furthermore, the quantity e_1 when added to the rate of flow of water q_s gives the total rate of runoff of water and soil combined. For illustration, lines showing $(q_s + e_1)$ have been added to the hydrographs for experiments numbers 9 and 20.

Measuring devices for stream-channels measure the total flow (solids plus fluid). Volume-concentration of suspended matter can advantageously be used in this case also. If w_1 is the weight of silt per unit-volume of flow, and G = specific gravity of suspended solids, then

$$C_{vol} = Gw_1/q \quad (21)$$

The total volume of suspended matter, S_1 , carried by a given measured volume of the total flow q per unit of time is, then

$$S_1 = qC_{vol} \quad (22)$$

The author has elsewhere shown that for various types of steady hydraulic flow (without bore-fronts or mud-flows) there is a definite relation of runoff-intensity q_s and average detention-depth δ_a near the outlet end of a runoff-plot [6]. This relation is expressed by the equation [7]

$$q_s = [(M + 1)/M] K_s \delta_a^M \quad (23)$$

This relation applies to some of Neal's experiments but to most of them it apparently does not apply. Three examples of the relation of q_s to δ_a are shown on Figure 9. Numbers indicate the sequence of the 10-minute intervals for which the values of q_s and δ_a were determined. In each instance the initial point (No. 1) indicates a flow rate q_s much below that for stable flow. This, combined with the abrupt fronts of many of the hydrographs, suggests that in these cases runoff started with a miniature mud-flow similar to the debris-dams often occurring at the fronts of cloudburst-floods in mountain canyons. The high frictional resistance of the mud-front cuts down the velocity much below that for normal hydraulic flow at the same depth. Figure 9 also shows that the discharge-rate q_s became nearly constant except for the first and second intervals and was thereafter independent of the depth δ_a of surface-detention. It will be noted that the points with the same q_s and δ_a occur in pairs for successive time-intervals. A condition of flow independent of depth can occur hydraulically in conjunction with a constant volume of flow passing through a reach of a stream in which a back-water curve exists. Back-water curves behind debris-dams in case of floods in canyons are not of uncommon occurrence.

The back-water curve in a permanent stream-channel remains stationary relative to an observer on the bank. If an observer travels along the stream and takes stage-, velocity-, and discharge-observations, he will find: (1) The volume of flow is constant; (2) there is no correlation of flow-rate to stream-stage; (3) since $q = \text{constant} = \delta wv$, if the width w is constant, then the velocity v will vary inversely as the depth δ . These conditions remain true even though the flow actually follows, for example, the Manning formula. The reason for these seeming anomalies is that the observations taken are not those required to reveal the true laws of flow.

In case of the back-water curve behind a mud-flow, a stationary observer taking observations as the back-water curve passes him will find exactly the same things. It is evident that in such cases the law of surface-runoff cannot be directly determined from the observations.

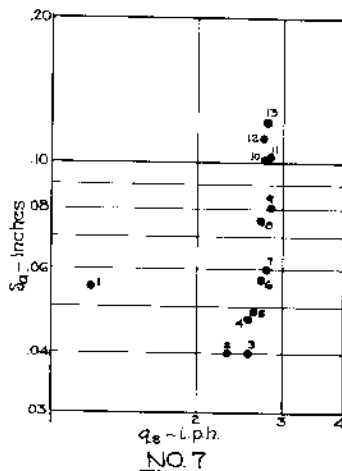
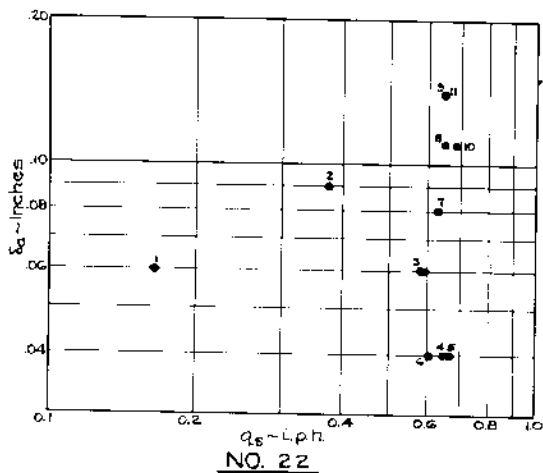
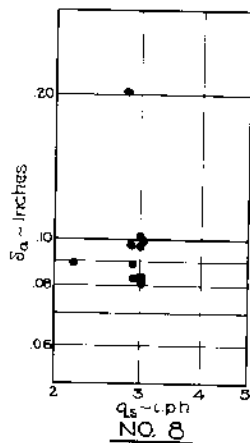
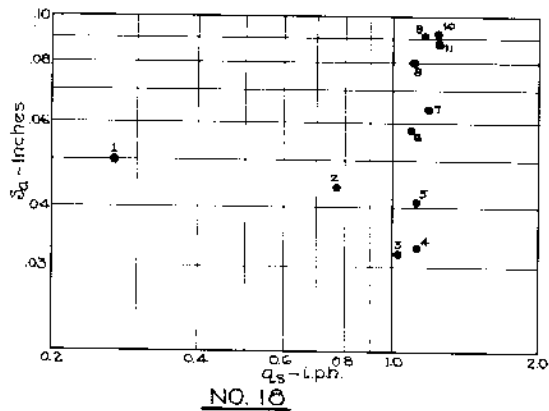


FIG 9--RELATION OF AVERAGE DETENTION-DEPTH TO RUNOFF-INTENSITY (WATER ONLY); NEAL'S EXPERIMENTS

The occurrence of points in pairs on Figure 9 indicates that the degradation of the soil-surface takes place by more or less abrupt jumps or quanta. In most cases the second of a pair of adjacent points shows both higher stage (δ_a) and larger flow (q_s). If two of these points are connected by a line, its slope will be found to be usually between 5/3 (fully turbulent flow) and 2.33 (50 per cent turbulent), indicating that, during a given stage, the flow was in accordance with equation (23). It appears probable that in Neal's experiments, lack of direct relationship of q_s to δ_a is, in part at least, the result of successive formation and breaking of debris dams or divides between depressions, accompanied by abrupt changes of δ_a without a corresponding change of q_s , due to successive stages of gullying. Even then the fact that δ_a progressively increased during a given experiment without a corresponding increase of q_s remains to be accounted for. This may be due to the fact that on the ungullied portion of the plat, the detention-depth remained nearly as great as without the gullies. As a gully forms, a high concentration of suspended matter builds up either a debris-dam near the outlet or a mud-flow in the gully, behind which back-water accumulates. This increases the average detention without a corresponding increase of q_s .

The hydrographs on Figures 6, 7, and 8 are plotted in terms of water-flow only. The abrupt fronts of many of the hydrographs may be due to initial mud-flow, with abrupt release of water as the mud passed the outlet sill of the runoff-plat. These abrupt fronts are, however, due in a considerable degree to the variation of infiltration-capacity f . A decrease of f with time has the same effect on the form of the hydrograph as a corresponding increase of rain-intensity with constant f .

The author has elsewhere given the semi-rational equation of the rising side of the hydrograph [8]

$$q_s = \sigma \tanh^M [(M+1)/M] (\sqrt[M]{\sigma K_s}) t \quad (24)$$

where σ = the supply-rate ($1-f$), K_s and M are the coefficient and exponent, respectively, in the surface-runoff equation (23), t is time in hours, and q_s the runoff-intensity in inches per hour. This equation is rational for 75 per cent turbulent flow ($M=2$) and closely approximative of other types of flow from 50 per cent laminar ($M=2.33$) to fully turbulent ($M=5/3$). The degree of turbulence is related to M through the equation

$$I = (3/4) (3.0 - M) \quad (25)$$

Equation (24) applies, however, only with a constant supply-rate σ . For an increasing supply-rate, as with a decreasing infiltration-capacity and constant rain-intensity, the supply-rate can be assumed constant and equal to its average value in each of a series of short successive time-intervals.

Let the supply-rates for interval 0 to t_1 , t_1 to t_2 , etc., be designated σ_1, σ_2 , etc. For interval 1, σ_1 in equation (24) gives the required values of q_s ending with q_1 at t_1 . If the supply-rate had been σ_2 from the start, then at the end of some time-interval t'_1 , q_s would equal q_1 and t'_1 can be determined from equation (24) by making $q_s = q_1$ for $t = t'_1$ and $\sigma = \sigma_2$, and solving for t'_1 , giving

$$t'_1 = [M/(M+1)] [1/\sqrt[M]{\sigma_2 K_s}] \tanh^{-1} (\sqrt[M]{\sigma_1/\sigma_2}) \quad (26)$$

If, now, q_s is computed for a series of times $t'_1 + \Delta_1$, $t'_1 + \Delta_2$, etc., to $t'_1 + (t_2 - t_1)$, using σ_2 in equation (24), the resulting values of q_s will be the correct values for the times $(t_1 + \Delta_1)$, $(t_1 + \Delta_2)$, etc. to t_2 . In this way the arcs of the hydrograph can be computed and filled in for successive time-intervals, with different supply-rates [1c].

Calculations have been made, as shown on Figure 10 of three runoff-graphs, with $K_s = 10$ and for 75 per cent turbulent flow. Line A shows the runoff-graph with the infiltration-capacity decreasing with duration of rain, in the manner shown by the average of Neal's experiments. Line B shows the runoff-graph with the infiltration-capacity constant and equal to the average value ($f = 1.26$ i.p.h.) shown by Neal's experiments. Line C shows the runoff-graph for the infiltration-capacity constant and equal to the average minimum value ($f = 0.166$ i.p.h.) shown by Neal's experiments. In all cases the rain-intensity $i = 4$ i.p.h. is constant. The area between lines A and C shows the reduction of surface-runoff resulting from a high initial infiltration-capacity as compared with the constant infiltration-capacity f_c . The ultimate runoff-intensity is, however, the same in both these cases. Lines A and B show that initially high infiltration-capacity materially reduced the runoff in the first 0.2 hour as compared with a constant infiltration-capacity equal to the average, but after 0.2 hour the infiltration-capacity fell below the average, and both the total runoff and the runoff-intensity are greater than for a constant infiltration-capacity.

A further point to be noted is that, as shown by line A, with varying infiltration-capacity the front of the hydrograph is much more abrupt than for an equal constant average infiltration-capacity. Line B differs, however, from most of Neal's experiments and many other experiments where erosion occurred, in that the front of the hydrograph B starts gradually with slope zero at time $t = 0$, whereas in Neal's and many other hydrographs where high initial erosion takes place, the front of the hydrograph has a vertical slope. This difference indicates at once that in these hydrographs the abrupt fronts are in the main due to mud-flow.

Soil-erosion--It is to be noted that the first measurement of erosion-rate was at the end of ten minutes' rain. Many of the experiments, for example, numbers 3, 4, 9, and 10, show maximum erosion-rates for the first 10-minute interval.

Erosion-concentration may be designated C_{vol} and defined as the ratio of solid material in the runoff to the total runoff, including both water and solid material, or, for a given time-interval

$$C_{vol} = e_1/(e_1 + q_s) \quad (27)$$

If only solid matter was carried away, q_s would be zero and C_{vol} would be unity. The erosion-

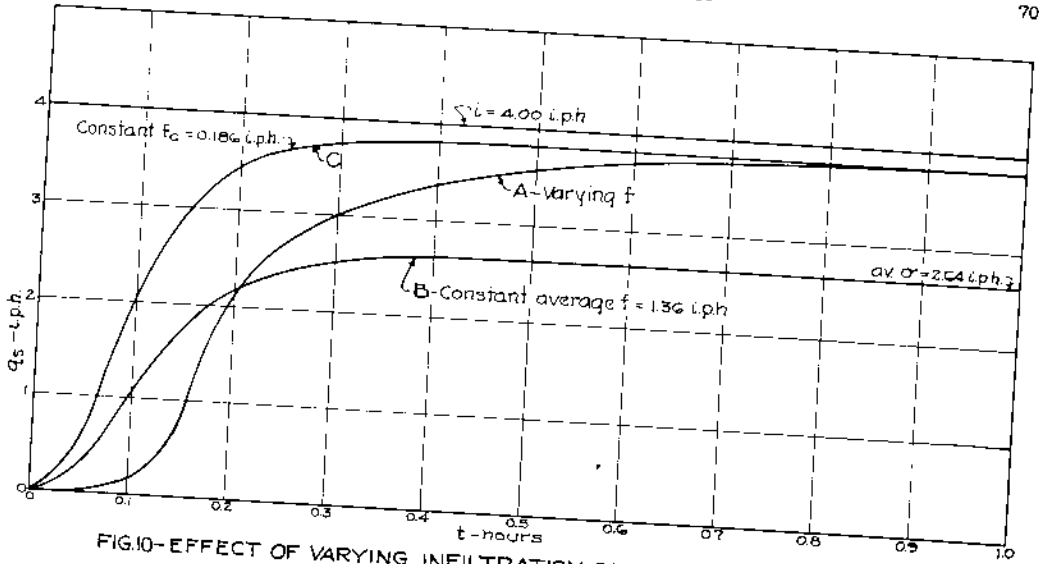


FIG. 10 - EFFECT OF VARYING INFILTRATION-CAPACITY ON RUNOFF-GRAPH

concentration for one of Neal's experiments is shown on Figure 11. The high initial concentration of solid matter suggests that in these experiments the initial loss of soil-material was largely due to rain-impact preceding or in the early stages of runoff--a condition compatible with the occurrence of miniature mud-flows.

Other experiments yield similar but not always consistent relation curves of C_{vol} to t . Erosion-concentration starts at a high value C_0 and approaches a constant value C_c asymptotically. The constants in the equation

$$C_{vol} = C_c + (C_0 - C_c)e^{-Kct} \quad (28)$$

can be evaluated by the same method as for infiltration-capacity curves in Figure 3. The identity of form of curve and form of equation for erosion-concentration-time and for infiltration-capacity-time (Fig. 2) suggests that reduction of erosion-concentration with duration of runoff is the result of one or more of the same factors which produce reduction of f . Rain-impact is probably responsible for the high initial concentration C_{vol} , but rain-packing and adhesion from

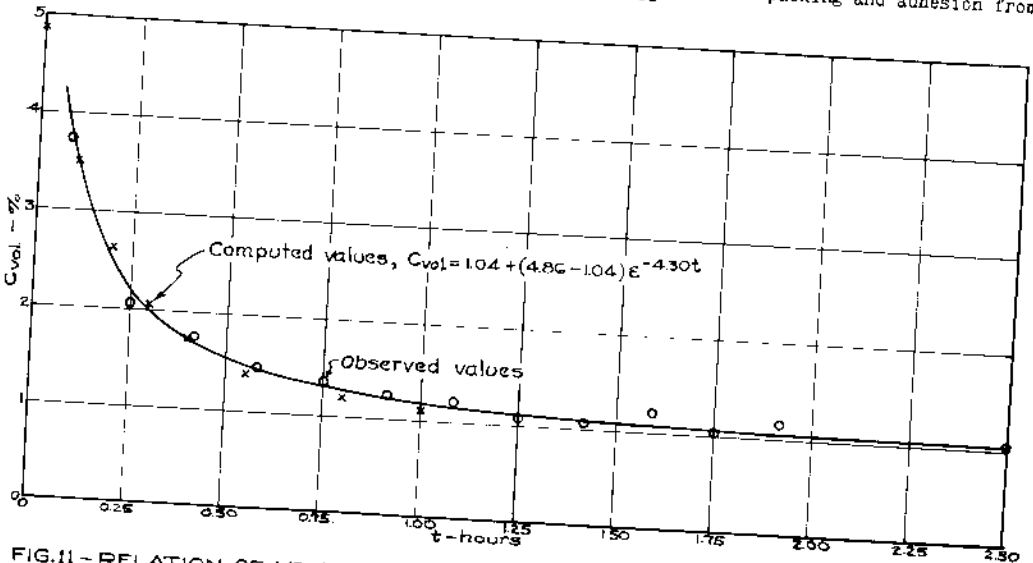


FIG. 11 - RELATION OF VOLUMETRIC CONCENTRATION OF SEDIMENT TO DURATION OF RAIN, NEAL'S EXPT. NO. 9.

TABLE 3-- SOIL-EROSION CONSTANTS -- NEAL'S EXPTS

No.	S	i	q _s	S _a	v	e ₁	e ₁ '	C _{vol.}
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
6	1	3.028	2.783	0.119	0.057	0.0096	0.0749	0.34
		1.453	1.172	0.59	0.46	0.020	0.027	1.7
		2.240	1.983	0.89	0.52	0.058	0.388	2.5
7	2	4.145	3.949	0.137	0.064	0.0388	0.9973	0.96
		3.028	2.768	0.83	0.72	0.133	0.19	4.8
11	2	3.998	1.687	0.42	0.90	0.0660	0.171	3.5
		1.494	1.267	0.98	0.29	0.022	0.035	1.7
16	2	2.666	2.418	0.92	0.64	0.149	0.1799	4.9
2	4	4.100	3.905	0.103	0.084	0.0497	0.7578	1.26
		3.032	2.977	1.00	0.70	0.247	2.189	8.2
8	4	2.085	1.949	0.53	0.86	0.080	0.304	4.1
		1.512	1.339	0.18	2.07	0.058	0.104	4.3
17	4	2.690	2.542	0.68	1.12	0.220	2.544	7.3
3	8	4.069	3.811	0.050	0.170	0.0760	1.1038	1.96
		2.952	2.719	1.22	0.53	0.341	2.521	1.24
9	8	2.016	1.651	0.08	0.69	0.215	0.586	1.28
		1.483	1.147	0.64	0.45	0.133	0.175	1.15
18	8	0.900	0.641	0.09	2.15	0.030	0.123	0.47
		2.284	1.994	0.51	1.10	0.296	2.889	1.22
19	12	1.502	1.247	0.068	0.64	0.0140	0.0218	1.11
4	16	4.057	3.601	0.038	0.216	0.1431	1.8556	3.83
		3.096	2.904	1.32	0.49	0.427	3.601	1.45
10	16	2.023	1.681	0.79	0.47	0.453	1.280	2.62
		1.485	1.234	0.83	0.23	0.187	0.285	1.49
20	16	2.665	2.355	0.83	0.86	0.624	5.930	2.35

(a) As given in Col. 6, $v = \frac{q_s}{S}$. These are relative quantities, not the true mean velocities, of the plot

intensity. The author finds

$$e_1 = S^{5/8} q_s^2 \quad (29)$$

Neal [1] found

$$E \text{ (pounds per acre per hour)} = 0.4 S^{0.7} i^{2.2} \quad (30)$$

The difference in the exponents 2.2 and 2.0 for i and q_s is probably due to the fact that for stable flow-conditions the infiltration-capacity was nearly constant and equal to f_c , so that $q_s = (1 - f_c)$.

Equations (29) and (30) are empirical and require confirmation before application to other soils or other conditions. This is especially true in view of the fact that in many of the experiments gullying occurred, and the equation may not be applicable to conditions of pure sheet-erosion without gullying. Exclusive of initial erosion due to rain-impact, there is no necessary relationship between rain-intensity and erosion-rate, and it seems obvious that Neal's equation cannot be applied to conditions where the infiltration-capacity was different from that in his experiments. For example, in Neal's experiments, with $i = 4.0$ i.p.h., q_s was approximately 3.9 i.p.h. For other soils with different infiltration-capacities, the same runoff-intensity might have resulted with widely different rain-intensities.

The question arises whether for a given soil and surface-conditions, erosion-rate can better be expressed in terms of the two variables, runoff-intensity q_s and slope S , or in terms of

wetting and swelling of colloids appear to be the main factors tending to reduce erosion-concentration. Thus rain-intensity appears to operate in two opposite ways: At the beginning of rain, high intensity induces high erosion-concentration but after a few minutes the increased rain-packing checks the erosion-rate and reduces the concentration. This is partly because of decrease of the infiltration-capacity f and consequent increased runoff-intensity q_s . In view of the open question as to how much of the observed soil-erosion was due to rain-impact and how much was due strictly to surface-runoff, it has seemed desirable to attempt to study these two effects separately. For this purpose Table 3 was compiled. This shows various factors for each experiment for the interval during which the runoff was sensibly constant, including the rate of soil-removal e_1 , the corresponding rate of flow, and the depth of surface-detention and velocity of overland-flow, both near the outlet end of the plot.

The author has made an analysis of the erosion-rate in terms of q_s , like that made by Neal in terms of rain-intensity i . This shows erosion-intensities varying as some power of q_s or i a little greater than the square. Logarithmic plotting in terms of q_s shows an exponent of q_s so close to 2 that the statement may fairly be made that for the conditions and within the limits of Neal's experiments, the erosion-rate for stable flow varies nearly as the square of the runoff-

detention-depth δ_a and velocity of overland-flow v_0 . The latter appear to be the more direct variables controlling erosion-rate; also they take into account the effect of slope. Probably because of gullying and the formation and breaking down of debris-dams, the author has not found a satisfactory relationship from Neal's experiments between either erosion-rate and depth of surface-detention or between erosion-rate and velocity of overland-flow.

This discussion of erosion is merely a preliminary approach to the analysis of this complex problem. It seems desirable that experiments primarily to determine infiltration-capacity f and the law of overland-flow should be segregated from those intended to determine erosion-phenomena. In the former the rain-intensity must exceed f but should be kept below that producing active erosion; in the latter the rain-intensity should be above the critical value to initiate erosion. But there are two cases which should be studied separately: (1) Pure sheet-erosion without gullying; (2) erosion where gullying occurs. This can in general be accomplished by the use of a rain-applicator capable of producing a sufficiently wide range of rain-intensities without too large gaps between successive intensities.

References

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- [8] Robert E. Horton, The interpretation and application of runoff plot experiments with reference to soil erosion problems, Proc. Soil Sci. Soc. Amer., v. 3, 1938.
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THE RELATION OF RAINFALL TO ELEVATION IN THE SOUTHERN APPALACHIAN REGION

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An investigation of the relation of elevation to the variation in annual rainfall at a number of rainfall-stations in western United States was made about 1899 by Mr. J. B. Lippincott [see 1 of "References" at end of paper]. This study was extended by Dr. Daniel W. Mead [2] in 1919 so as to include additional stations in Europe and the United States. These investigations indicated rather conclusively that in most mountain areas the amount of mean annual rainfall was greater, the higher the elevation of the station considered.

At the time these studies were made, practically no data were available for the higher sections of the Appalachian Mountains, and this region could not be included in the areas considered. Collection of rainfall-data for the higher parts of the Southern Appalachian Region was inaugurated by the Tennessee Valley Authority in 1934 as a part of its investigation of the water-resources of the Tennessee River Basin and is still being carried on by that agency.

There are now 84 rainfall-stations located at or above an elevation of 2500 feet above mean sea-level, and continuous records are available for 73 of these for the period October 1, 1935, to September 30, 1938. These records have been carefully analyzed with a view to determining what relation exists in this Southern Appalachian Region between the amount of mean annual rainfall and the elevation of the stations at which it is recorded. The results of this analysis are presented herewith.

Geographical location of the Region--The upper half of the Tennessee River Basin drains about 10,000 square miles of the Great Smoky and the Blue Ridge Mountains lying in Virginia, North Carolina, Tennessee, and Georgia. These mountains extend in a general direction from