Comment on "On the continuum-scale modeling of gravity-driven fingers in unsaturated porous media: The inadequacy of the Richards equation with standard monotonic constitutive relations and hysteretic equations of state" by Mehdi Eliassi and Robert J. Glass

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Received 21 October 2002; accepted 4 April 2003; published 18 September 2003.

INDEX TERMS: 1875 Hydrology: Unsaturated zone; *KEYWORDS:* unsaturated zone, fingering, wetting front, instability

Citation: Deinert, M., J.-Y. Parlange, K. B. Cady, T. S. Steenhuis, and J. S. Selker, Comment on ''On the continuum-scale modeling of gravity-driven fingers in unsaturated porous media: The inadequacy of the Richards equation with standard monotonic constitutive relations and hysteretic equations of state'' by Mehdi Eliassi and Robert J. Glass, *Water Resour. Res.*, *39*(9), 1263, doi:10.1029/2002WR001785, 2003.

[1] This comment concerns the discussion by *Eliassi and Glass* [2001] of the applicability of Darcy's law and Richards's equation in describing fingered flow. *Nieber* [1996] was the first to model experimentally observed unstable fingered flow successfully. In order to solve Richards's equation, Nieber used a weighting parameter, *w*, to adjust the hydraulic conductivity at the wetting front. As w nears -1, the flow simulations mimic instabilities, which have been observed as nonviscous fingers in soils [e.g., *Hill and Parlange*, 1972]. Taking different weighting parameters adjusts the maximum water content and width at the tip of the simulated finger, thereby allowing the fit to what is, in fact, observed. However, this weighing parameter should not be seen as the proper representation of some physical process.

[2] Indeed, any continuum approach, for example, Richards's equation or Darcy's law, is problematic at the wetting front where pore-scale processes are taking place. This is obvious from pore-scale visualization by, among others, *Lu et al.* [1994], which shows water jumping from one pore to the next in distinct steps. For this reason, in their measurements of finger structure and the subsequent discussion based on Darcy's law, *Selker et al.* [1992] and *Liu et al.* [1994] were careful to consider the drying part of the fingers where water content varied slowly. Use of any expression related to Darcy's law assumes that gradients are taken over Darcy's scale, which must involve a sufficient number of pores. Flow history is also crucial [*Liu et al.*, 1994], and the maximum water content at the finger's tip was obtained experimentally, as it could not be predicted from Darcy's law.

[3] Further insight is obtained from the fingered flow experiments of *Deinert et al.* [2002]. Using neutron radiography, *Deinert et al.* [2002] were able to measure reliable water contents down to the pore scale. Since measurements of pressure with a standard tensiometer are not reliable during rapid wetting, they defined a "dynamic" pressure, h, for a wetting front moving at constant velocity, v, using a Darcy-like expression to define $h(\theta)$ mathematically as

$$v\theta = k(\theta)\frac{dh(\theta)}{d\xi} + k(\theta), \tag{1}$$

where $k(\theta)$ is the standard soil-water conductivity, measured with a multistep experiment [*Deinert et al.*, 2002] as a function of the water content, θ, and $\xi = vt - z$, where z is positive downward. With v, θ and $k(\theta)$ measured reliably, equation (1) yields $h(\theta)$ as shown in Figure 1 (as only the gradient of h enters Darcy's law, we did not obtain the exact position of $h = 0$, and an irrelevant, small translation of the h scale is possible). However, it must be reiterated that h in equation (1) must be seen as a mathematical entity and not as a physically defined property, as Darcy's law cannot be written for pore-scale phenomena, as discussed earlier.

[4] *Eliassi and Glass* [2001] mention that the work of *Gray and Hassanizadeh* [1991] is germane to understanding the nonequilibrium wetting process. Other later work by *Hassanizadeh and Gray* [1993], *Beliaev and Hassanizadeh* [2001], and *Beliaev and Schotting* [2001] as well as the review by *Hassanizadeh et al.* [2002] suggests thermodynamic approaches to understand and model this phenomenon. Using those nonequilibrium concepts, *Dautov et al.* [2002] were able to simulate flow instability and finger formation by assuming a "dynamic" pressure very much like the one shown in Figure 1. Of course, the fact that the mathematical $h - \theta$ relationship shown in Figure 1 is consistent with instability does not necessarily imply that it is due to nonequilibrium effects. It could very well be due to some other process, for example, strong hysteretic effects associated with contact angle changes in infiltration [*Selker*

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0043-1397/03/2002WR001785$09.00

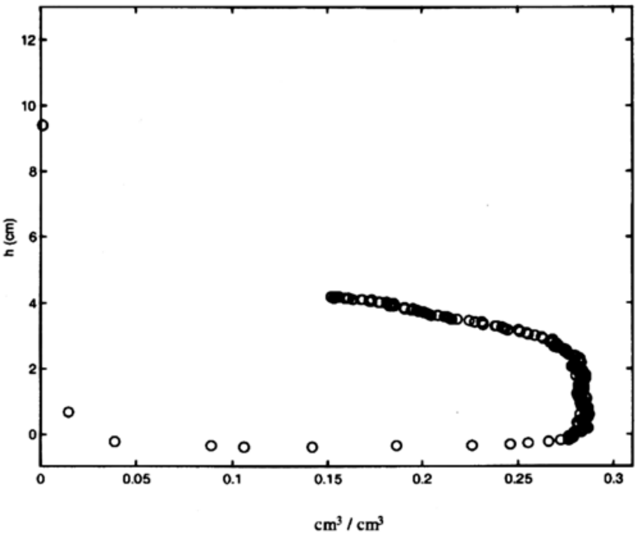

Figure 1. The "dynamic" pressure obtained experimentally using equation (1) [*Deinert et al.*, 2002].

and Schroth, 1998]. In any case, the important result of *Dautov et al.* [2002] remains that an $h - \theta$ relationship similar to the one shown in Figure 1, which results from the measurements of *Deinert et al.* [2002], leads to instability when used in conjunction with a continuum approach.

[5] In conclusion, the critique by *Eliassi and Glass* [2001] of *Nieber* [1996] is somewhat exaggerated since the results of Nieber's simple model are consistent with observations. In Nieber's early work on numerical simulation of fingers the *w* factor was only used as a fitting parameter to mimic observed fingering. In any case, and more importantly, the mathematical $h - \theta$ relationship as shown in Figure 1 is consistent with instability. Whether this observed relationship is the result of nonequilibrium or of some other phenomenon is still unanswered, but in any case, it must be emphasized that such an h, obtained at the wetting front, is only defined as a mathematical entity. Clearly, one would like to solve the Navier-Stokes equations at the wetting front within individual pores. However, even then, assumptions about contact angles would be required. Macroscopic-type modeling as discussed by *Nieber* [1996], *Eliassi and Glass* [2001], and *Dautov et al.* [2002] leads to interesting discussions and speculations, but what is required are experimental observations at the pore scale.

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