Preferential Flow of a Nonaqueous Phase Liquid in Dry Sand

P. J. Culligan, M. ASCE; K. Banno; D. A. Barry; T. S. Steenhuis; and J.-Y. Parlange

Abstract: Geotechnical centrifuge testing is used to examine the preferential (fingered) flow of a nonaqueous phase liquid (NAPL) in a uniform dry sand. The results of nine experiments, containing a total of 87 observations of NAPL finger behavior, are analyzed. The observed finger tip velocities range from 0.01 to 0.3 cm/s, while the observed finger widths range from 0.3 to 3.6 cm. From the experimental data it is concluded that, asymptotically, the NAPL fingers are not fully saturated. For comparison, the behavior of water fingers is examined using the same experimental setup. In contrast to the NAPL fingers, and in agreement with other work reported in the literature (e.g., Glass et al. 1989), the water fingers are found to be fully saturated. In addition, it is confirmed that the water finger properties can be well predicted from known porous medium and fluid properties. A scaling analysis is presented that allows the NAPL finger properties to be inferred from models developed to describe water finger properties. The analysis predicts NAPL finger velocities to within 15% and NAPL finger widths to within 50% if both finger types are assumed saturated. By adjusting the analysis to account for the fact that the NAPL fingers are not fully saturated, NAPL finger widths can be predicted within to 10%, and NAPL finger velocities to within 30%.


Introduction

Previous belief that the area above the water table—called the vadose zone—is an effective barrier between hazardous waste and the groundwater, has led to the siting of numerous waste disposal facilities in regions where the vadose zone is 70 m thick or higher (e.g., the Hanford Site, Wash.). Often, the aquifers in these regions are composed of dry, unsaturated rock, such as is the case at the Yucca Mountain Test Site, Nev. (Mielke 1999), or dry unsaturated sand, such as is the case at Massachusetts Military Reservation, Mass. (LeBlanc et al. 1986).

Since the early 1980s, belief that the unsaturated vadose zone provides an effective barrier for waste transport has been shaken by innumerable reports of the unexpected, rapid transport of chemicals through this zone to the water table (Tindall and Kunkel 1999). In many instances, these reports can be explained by the presence of so-called preferential flow, which is the movement of fluids through preferred pathways in soils. During preferential flow, small areas of the unsaturated subsurface act as conduits for fluid flow, leading to the rapid, and uneven, movement of fluids and related chemicals to the groundwater. In heterogeneous soils, the phenomenon of preferential flow can be attributed to funneling, which is the directed flow of infiltrating fronts by heterogeneous soil structures (Kung 1990), and/or channeling, which is the transport of fluid through cracks and macropores (Tindall and Vencill 1995). However, observations of the unexpected, rapid transport of chemicals though unsaturated soils have also been made in homogeneous geological media (Starr et al. 1985).

The preferential flow of an infiltrating fluid in a dry, homogeneous medium is caused by instabilities generated at the front of the infiltrating fluid. This form of preferential flow is often termed fingering. In unsaturated soils, the infiltrating fluid is denser and more viscous than the interstitial fluid (air), and normally preferentially wets the solid phase of the medium (Bear 1972). Under these circumstances, fluid fingers are generated by gravity-driven wetting front instability (Hill 1952). Extensive studies of gravity-driven wetting front instability in dry soil zones have been carried out for cases where water is the infiltrating fluid (e.g., Hill and Parlange 1972; Diment and Watson 1985; Liu et al. 1994). These studies have shown that preferred flow pathways are generated when the infiltrating fluid flux is insufficient to fully saturate the medium. This condition can occur with uniform infiltration at low application rates (e.g., DiCarlo et al. 1999) or when there is an increase in the hydraulic conductivity of the medium with depth; for example, when a fine-textured soil overlies a coarser layer (e.g., Culligan et al. 1997). In addition, these studies have demonstrated that many of the water finger properties during preferential flow can be predicted from known porous medium and fluid properties (e.g., Glass et al. 1991; Griffioen and Barry 1999).

The objective of this work is to examine whether some of the approaches for predicting gravity-driven wetting front instability of water in unsaturated soils can be extended to a nonaqueous phase liquid (NAPL). NAPLs, such as gasoline and Trichloroethylene, represent 16 of the 25 most frequently detected contami-
namentals at hazardous waste sites in the United States (US EPA 1994). Often these liquids enter the subsurface as spills from storage tanks located at the soil surface or buried within the vadose zone itself (Russell et al. 1991). In situations where the vadose zone is extensively comprised of a homogeneous, dry soil, such as is the case at the Massachusetts Military Reservation, Mass., fingered flow of these liquids during downward transport is possible if the NAPL spill is insufficient to saturate the underlying strata. Under these circumstances, predictions of NAPL finger behavior are a necessary precursor to the planning of an effective remedial action.

This paper describes an experimental program that made use of the technique of geotechnical centrifuge testing to examine two-dimensional NAPL finger behavior in a uniform, dry sand under a wide range of infiltration conditions. During each experiment, the finger dimensions, the finger spacing, and the velocity of finger propagation were measured. For comparison, the behavior of water fingers using the same experimental setup was also observed. Independent laboratory measurements of the fluid and porous medium properties were then used to predict the behavior of the water fingers, and to investigate scaling relationships between the water finger properties and the NAPL finger properties.

Background

Gravity-Driven Wetting Front Instability

During infiltration of a wetting fluid into a homogeneous porous medium, the wetting front will encounter inherent microscopic heterogeneities that cause small perturbations in the infiltrating front. If the forces acting to destabilize the flow dominate the forces acting to stabilize it, then a finger will develop from this perturbation (Kueper and Frind 1988). For downward flow when the displacing fluid is denser and more viscous than the displaced fluid, gravitational (body) forces are destabilizing. For the special case where the density and viscosity of the displaced fluid are negligible in comparison to the density and viscosity of the displacing fluid, the necessary (but not sufficient) condition for finger growth is given by (Hill 1952)

\[ q_i \leq \frac{k_F g \rho}{\mu} \]  

(1)

where \( q_i \) is the flux of infiltrating fluid through the system, \( k_F \) is the fluid permeability of the medium at the wetting front, \( \rho \) and \( \mu \) are the density and viscosity of the infiltrating fluid, respectively, \( g \) is the gravitational acceleration, and \( k_F g \rho / \mu \) is the fluid hydraulic conductivity \( K_F \) at the wetting front.

Following the onset of fingering in a uniform soil, the macroscopic parameters most frequently used to describe the preferential flow include the finger width \( w \) (or the finger diameter \( d \) in the case of three-dimensional flow), the finger spacing \( \lambda \) and the finger velocity \( u \). This study confined itself to the investigation of the two-dimensional fingered flow described by Fig. 1, and adopted the following approximations for \( w \) and \( u \) (Parlange and Hill 1976), respectively,

\[ w = \frac{\pi S^2}{K_F(\theta_F - \theta_0) \left[ 1 - \frac{q_i}{K_F} \right]} \]  

(2)

and

\[ u = \frac{q_F}{(\theta_F - \theta_0)} \]  

(3)

where \( S = \) the infiltrating fluid sorptivity, \( \theta_F = \) the average volumetric fluid content of the finger, \( \theta_0 = \) the initial volumetric fluid content of the medium (zero in the case of an initially dry soil), and \( q_F = \) the average fluid flux through the finger; the magnitude of which is given by Darcy’s law as

\[ q_F = \left| K_F \left( \frac{\partial h}{\partial z} + 1 \right) \right|, \]  

(4)

where \( h = \) the matric potential and \( z = \) the length of the finger (see Fig. 1).

The sorptivity quantifies the effect of capillarity on water movement in soil. The parameter was first proposed by Philip (1957) who, for the condition of fluid infiltration into initially dry soil, described it as

\[ S = \left( \frac{\sigma \cos \phi}{\mu} \right)^{1/2} s, \]  

(5)

where \( \sigma = \) the surface tension of the fluid, \( \phi = \) the contact angle of the fluid, and \( s = \) the intrinsic sorptivity of the medium, a parameter dependent on the geometry of the medium and the boundary conditions alone. Like the intrinsic permeability of a medium, which is generally inferred from measurements of the fluid-dependent hydraulic conductivity, \( S \) is generally inferred from measurements of \( S \).

The ratio \( q_i / K_F \) is the system flux ratio \( R \). From Eq. (2), if \( R = 1 \) (i.e., \( q_i = K_F \)), the fingers will be infinitely wide. In other words, if the infiltrating fluid saturates the medium uniform rather than preferential flow will occur, which is simply the condition given by Eq. (1).

As the finger length grows, the hydraulic gradient at the finger tip will approach unity (Selker et al. 1992). Therefore, \( q_F \) will asymptotically approach \( K_F \) [see Eq. (4)]. By conservation of mass, \( w q_F = \lambda q_i \). Therefore, the finger–finger separation can be approximated as

\[ \lambda = \frac{w K_F}{q_i} \]  

(6)

Geotechnical Centrifuge Testing

During geotechnical centrifuge testing, a container housing a soil experiment is spun around a central axis at a high rotational speed, thereby increasing the body forces acting on the soil and fluids within the container. At any given rotational speed \( \omega \), the experiment will experience a centrifugal acceleration \( r \omega^2 \), where \( r \) is the radial distance of the center of gravity of the container from the central axis. In the centrifuge modeling community, it is customary to describe this acceleration as the product of the Earth
gravitational acceleration $g$ and a scaling factor $N$—often termed the $g$ level (Taylor 1995).

Consider an experiment that is designed to produce the two-dimensional finger pattern shown in Fig. 1. If the experiment is conducted at a centrifugal acceleration that is equivalent to $Ng$, then the hydraulic conductivity of the soil to the infiltrating fluid at the wetting front will be given by

$$K_{F(Ng)} = \frac{k_F(Ng) \rho}{\mu}. \tag{7}$$

The permeability $k_F$ is the product of $k_0$, the intrinsic permeability of the medium and $k_r$, relative permeability of the infiltrating fluid in the medium at a volumetric fluid content of $\theta_F$. For a nondeforming porous medium, $k_0$, $\rho$, and $\mu$ will not change with $N$, $k_r$ is a function of $h_F$, the mastic potential at the finger tip. Hence, $k_0$ will not change with $N$, and neither will $s$, $\sigma$, and $\phi$, indicating that $S$ will also remain invariant with $N$ [see Eq. (5)]. However, because the average finger flux $q_F$ is directly proportional to $K_F$ [see Eq. (4)], $q_F$, like $K_F$, will be $N$ times higher at $Ng$ than at $g$.

The method of geotechnical centrifuge testing therefore offers a convenient means of investigating the phenomenon of gravity-driven wetting front instability over a range of soil hydraulic conductivities. This can be achieved simply by subjecting the same experimental setup to different centrifugal accelerations, and observing the fingering properties formed at each $g$ level. An alternative to the use of the geotechnical centrifuge would be to vary $k_F$, $\rho$, and/or $\mu$ between experiments [see Eq. (7)]. This would involve constructing a series of experiments using different soils and/or infiltrating fluids. Although feasible, neither of these alternatives was considered as convenient to the use of the geotechnical centrifuge.

For the work reported here, the experiments were engineered to produce a system flux ratio, $R = q_F/K_F$, that did not vary with $N$ (see Experimental Methodology). Rewriting Eqs. (2), (3), and (6) for an experiment that is being conducted at constant $R$ under a centrifugal acceleration of $Ng$ gives

$$w_{(Ng)} = \frac{\pi S^2}{NK_F(\theta_F - \theta_0)(1 - R)} = \frac{w_{(1g)}}{N}; \tag{8}$$

$$u_{(Ng)} = \frac{Nq_F}{(\theta_F - \theta_0)} = Nu_{(1g)} \tag{9}$$

and

$$\lambda_{(Ng)} = \frac{w_{(Ng)}}{R} = \frac{\lambda_{(1g)}}{N}. \tag{10}$$

Thus, with increasing $N$, and hence increasing $K_F$, the width and the spacing of the fingers that are formed during preferential flow in a uniform soil will decrease, while the velocity of propagation of the fingers will increase. In the experiments reported here, the factor $N$ was varied from 1 to 11. This enabled information about NAPL finger properties to be gathered over an order of magnitude change in $K_F$.

### Preferential Flow of Water in Dry Soil

For the preferential flow of water into an initially dry soil ($\theta_0 = 0$), it has been observed that $K_F \approx K_{sw}$, where $K_{sw}$ is the water-saturated hydraulic conductivity of the medium (Glass et al. 1989). Thus, in the case of water infiltration Eqs. (2), (3), and (6) can be rewritten, respectively, as

$$w_w = \frac{\pi S^2}{K_{sw}(\theta_0 - 1)} \tag{11}$$

$$u_w = \frac{K_{sw}}{\theta_0} \tag{12}$$

and

$$\lambda_w = \frac{w_w}{R_w} \tag{13}$$

where the subscript $w$ = the infiltrating fluid water, and the subscript $s$ = fluid saturation. Note that $\theta_0$ is the saturated volumetric water content of the soil, which will generally be the porosity of the soil.

Equations (11), (12), and (13) can be used to predict the properties of the fingers formed during the unstable infiltration of water into a dry soil, if the water infiltration rate $q_{iw}$, is known, together with the medium properties $K_{sw}$, $\theta_0$, and $S_{sw}$, each of which can be determined from independent laboratory (ASTM 1997) and/or field (Lambe and Whitman 1979) tests.

### Scaling Analysis

A hypothetical relationship between the properties of water fingers formed in unsaturated soil, and the properties of NAPL fingers formed in the same soil, can be found from a simple scaling analysis, if it is assumed that both finger types are saturated at the finger tip. Note, that when a finger is saturated at its tip, $k_F = k_F$, the intrinsic permeability of the medium.

For fingers forming in the same soil, and under the same boundary conditions, $k_F$, $s$, and $\theta_0$, will be independent of the properties of the infiltrating fluid. Therefore, from Eqs. (1) and (5)

$$K_{sn} = \frac{P_s \mu_n S_{sn}}{P_w \mu_w S_{sw}} K_{sw}, \tag{14}$$

and

$$S_{n}^2 = \frac{\sigma_s \cos \phi_w \mu_w}{\sigma_w \cos \phi_w \mu_n} S_{sw}^2. \tag{15}$$

where the subscript $n$ denotes the fact that the infiltrating fluid is a NAPL.

Rewriting Eqs. (11), (12), and (13) for the case where NAPL is the infiltrating fluid, using Eqs. (14) and (15) to substitute for $K_{sn}$ and $S_{n}^2$, and rearranging gives

$$w_w = \frac{\sigma_{sw} \cos \phi_w \rho_w g_w (1 - R_w)}{\sigma_{sw} \cos \phi_w \rho_n g_n (1 - R_n)} \tag{16}$$

$$u_w = \frac{P_n \mu_n g_n}{P_w \mu_w g_w} u_w \tag{17}$$

and

$$\lambda_w = \frac{w_w}{R_w} \tag{18}$$

As noted in Preferential Flow of Water in Dry Soil the water finger properties in an unsaturated medium $w_w$, $u_w$, and $\lambda_w$ can be predicted for any $R_w$ using measurable medium properties. If the NAPL infiltration rate and NAPL fluid properties are known, then Eqs. (16), (17), and (18) can be used to estimate how NAPL fingers are likely to behave in the same unsaturated medium.
Table 1. Properties of SOLTROL 220 and Water at 20°C

<table>
<thead>
<tr>
<th>Property</th>
<th>SOLTROL 220</th>
<th>Water</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dynamic viscosity, ( \mu ) (g/cm³)/s</td>
<td>1.002 \times 10^{-2}</td>
<td></td>
</tr>
<tr>
<td>Density, ( \rho ) (g/cm³)</td>
<td>0.998</td>
<td></td>
</tr>
<tr>
<td>Surface tension, ( \sigma ) (dyne/cm)</td>
<td>71.9</td>
<td></td>
</tr>
<tr>
<td>( \cos \phi ), polished glass</td>
<td>0.99</td>
<td></td>
</tr>
<tr>
<td>( \cos \phi ), polished silica</td>
<td>0.86</td>
<td></td>
</tr>
</tbody>
</table>

*Properties for SOLTROL 220 were obtained from manufacturer's tables.

Experimental Methodology

The investigation of two-dimensional NAPL finger behavior in unsaturated soil was conducted using the aliphatic oil, SOLTROL 220, as the infiltrating fluid. To obtain finger properties during water infiltration, distilled water was used as the infiltrating fluid. Table 1 summarizes the properties of SOLTROL 220 together with the relevant properties of the water.

All experiments were carried out in a container, referred to as a centrifuge strongbox, which was constructed of T6061 aluminum plates and a Lexan transparent window through which images of flow patterns could be recorded (Fig. 2). The plates and window were assembled with stainless steel screws and sealed with silicone rubber so that there would be no fluid leakage during an experiment. The internal volume of the centrifuge strongbox itself was divided into three sections: the sample box, the fluid supply box, and the pump box. The sample box was used to hold the soil experiment. It had dimensions of 30.5 cm x 22.9 cm x 2.3 cm. The width, 2.3 cm, was selected in order to facilitate cleaning the box. However, during a test this dimension was reduced to 1.5, 1.1, or 0.8 cm using spacers of Plexiglas. The fluid supply box, which had a volume capacity of 10,000 cm³, was used to store the infiltrating fluid. On average, 500 cm³ of infiltrating fluid was required for each test. The pump box was used to hold a peristaltic pump that was employed to move fluid from the supply box to the sample box. A slot cut in the top edge of the aluminum plate that divided the sample box and the fluid box, allowed a fixed depth of ponded liquid to be maintained above the soil specimen contained in the sample box. Three 5-mm-diameter holes, cut 1.5 cm above the base of the aluminum plate that divided the sample box and the pump box, allowed any fluid in the sample box to drain freely to the atmosphere.

Fig. 3 shows a cross section through the soil experiment. During each test, 1.0 cm of fluid was maintained above a specimen composed of three soil layers. The top layer of soil consisted of uniformly packed 100–140 quartz sand (where 100–140 refers to the range of sieve sizes through which the sand passed; the metric equivalent of this range is 150–106 μm). The middle layer of soil consisted of uniformly packed 25–40 quartz sand (metric equivalent 775–425 μm). The bottom layer of soil consisted of uniformly packed quartz sand obtained from material left on the No. 25 sieve after sorting for the 25–40 sand. In all cases, the sands were washed with tap water and a detergent, rinsed a dozen times with distilled water, and then air dried over several days at room temperature before sorting and packing. To ensure uniform packing of the soil specimen, each sand layer was rained into the sample box from a fixed height and at a uniform rate. The height and rate for raining were selected to secure a void ratio for each sand that was close to its minimum void ratio. This minimized settlement, i.e., deformation, of the specimen during subsequent handling and testing. Table 2 summarizes properties for the 100–140 and 25–40 sands.

Table 2. Properties of 100–140 and 25–40 Sands

<table>
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<tr>
<th>Property</th>
<th>100–140 Sand</th>
<th>25–40 Sand</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average particle size, ( d_{50} ) (mm)</td>
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</tr>
<tr>
<td>Saturated conductivity, water ( K_{sw} ) (cm/s)</td>
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<td></td>
</tr>
<tr>
<td>Saturated conductivity, LNAPL ( K_{sw} ) (cm/s)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Brooks–Corey ( n )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Irreducible fluid content, ( \theta_i )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Water sorptivity, ( S_w ) (cm²/s)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*From constant head test (ASTM 2434-68).

Fig. 2. Centrifuge strong box

Fig. 3. Cross section through soil experiment
By maintaining a fixed, ponded depth of fluid on the surface of the soil specimen, the hydraulic gradient through the upper soil layer was kept invariant with the $g$ level $N$. This meant that the flux through the upper soil layer increased with $N$, as per the hydraulic conductivity of the underlying soil layer (see Geotechnical Centrifuge Testing). In this way, the ratio $R$ was kept similarly invariant with $N$. Because the flux through the upper soil layer was deliberately made insufficient to saturate the underlying, middle soil layer, fingers were expected to form as the infiltrating fluid passed the interface between these two layers. For this reason, the interface between the upper and middle soil layers was selected as the origin $z = 0$ for the measurements of the finger properties (Fig. 3).

All experiments were conducted using the balanced arm geotechnical centrifuge facility in the Department of Civil and Environmental Engineering at the Massachusetts Institute of Technology, (MIT), Mass. The MIT balance arm centrifuge is a Genisco Model 1231 G accelerator with a load capacity of 13,620 G kg. The centrifuge has a radius of 1.0 m and a maximum rotational speed of 400 rpm. Fig. 4 illustrates the experimental setup during a test. The strong box was bolted onto one of the centrifuge swinging platforms and a miniature, wide angled charge coupled device (CCD) camera was placed in front of the Lexan window of the box to provide a continuous record of each experiment. A 1 cm $\times$ 1 cm transparent reference grid was attached to the Lexan window so that finger properties could be extracted from the video record. A shower device, made of a 2.54-cm-diameter sealed aluminum tube perforated with seven, equally spaced 1-mm-diameter holes, distributed fluid supplied by the pump to the surface of the soil specimen.

During a typical experiment, image recording via the CCD camera was started and the angular rotation of the centrifuge was increased in stages until the test $g$ level was reached. The peri-staltic pump was then activated and stable infiltration of fluid under a fixed, ponded depth was observed in the upper layer of the soil model. When the infiltrating fluid reached the boundary between the upper and middle layers of the model, observation of finger development began. The test concluded when every single finger had either reached the base of the sample, or had stopped traveling for more than a few minutes. At this stage, the peri-staltic pump (Fig. 4) was stopped and the centrifuge speed was reduced to zero in seconds in order to conserve the finger patterns. When the centrifuge came to rest, the image recording was halted. Data relating to the final finger width and separation were then collected by visual inspection from the model. Finally, the images taken during the test were analyzed to obtain the finger properties during actual infiltration.

### Results and Discussion

In total, nine experiments were carried out to investigate unstable NAPL infiltration in the soil specimen. Within these nine experiments, 87 observations of finger behavior were made. Experiment 9 is a close replicate of Experiment 2. In addition, one experiment was carried out to investigate unstable water infiltration under the same conditions. A summary of experimental data, together with relevant information about the test settings, is given in Table 3. As predicted by Eqs. (8), (9), and (10), both $w$ and $\lambda$ decreases with an increasing $N$, while $\omega$ increases.

All finger properties were defined making use of the reference grid attached to the Lexan window. Corrections for the camera angle distortion and the thickness of the window were accounted for before any measurements were made. The number of fingers reported in each experiment is the number of fingers that reached the base of the soil specimen. The average finger width and finger separation are the average of all of these finger widths and separations at a depth $z = 100$ mm in the soil specimen. These properties were recorded at the end of each experiment. This reference depth was chosen because it was observed that the finger proper-

<table>
<thead>
<tr>
<th>Exp. No.</th>
<th>$N$</th>
<th>Fluid</th>
<th>$K_{PF}$ (mm/s)</th>
<th>$\theta_s$</th>
<th>Specimen width (mm)</th>
<th>No. of fingers</th>
<th>$w_{av}$ (mm)</th>
<th>$u_{av}$ (mm/s)</th>
<th>$\lambda_{av}$ (mm)</th>
<th>$R_{av}$ ($=w_{av}/\lambda_{av}$)</th>
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<td>0.37</td>
<td>15.5</td>
<td>0.37</td>
<td>0.62</td>
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</table>

Table 3. Summary of Experimental Settings and Measured Finger Properties. Average Finger Properties are Arithmetic Mean of all Observations. Deviation of Finger Properties is Absolute Deviation of all Observations from Their Mean.
ties did not change significantly beyond \( z = 100 \text{ mm} \). The average finger velocity is the average velocity of all of the fingers as they traversed between \( z = 100 \text{ mm} \) and \( z = 120 \text{ mm} \). This information was taken from the video recording made during each experiment.

From the data presented in Table 3, it can be noted that the maximum deviation of the observed finger widths from their mean is 20%, while the maximum deviation of the observed finger velocities and spacings are 25 and 38%, respectively. In their experiments of unstable water infiltration into dry soil, Griffen and Barry (1999) report maximum deviations of observed finger widths from their mean of 42%, and maximum deviations of observed finger velocities and spacings of 40 and 33%, respectively. Therefore, the amount of scatter observed in these experiments is typical of the scatter generally reported in fingered flow experiments.

Comparing the results of Experiment 9 with Experiment 2, it is seen that the average finger width observed in Experiment 9 is 8.5% higher than the average finger width observed during Experiment 2, while the average finger spacing is 2% higher. Therefore, the observations of \( w \) and \( \lambda \) appear to be fairly repeatable between experiments. However, the average velocity of fingers reaching the base of the soil specimen during Experiment 9 is 20% lower than the average velocity of fingers reaching the base of Experiment 2. This increased variation can be explained by the poor quality of the video images taken during Experiment 2, which made the postexperimental analysis of the finger velocities difficult, and therefore less precise.

Fig. 5 illustrates typical finger patterns that were developed during one of the tests. Note that, as in other studies of unstable water infiltration (e.g., Selker et al. 1992), both splitting and coalescing of some fingers, as well as the disappearance of others, was observed.

**NAPL Finger Behavior**

In general, the NAPL fingers were found to have a relatively uniform width with both depth and time. This observation was also made by Glass et al. (1989) during their study of water infiltration in dry sand. In addition, it was noticed that the well-defined fingers tended to move with a constant velocity, as shown by Fig. 6.

From Table 3, the average ratio of finger width to separation for Experiments 1–9 is given by \( w/\lambda = 0.62 \pm 0.16 \). Thus, the average value of \( R_s (= q_{in}/K_F, \text{ the system flux ratio}) \) is also 0.62 [see Eq. (6)].

Fig. 7 shows a plot of \( u_s\theta_s/K_{sn} \) versus the average finger width, where \( K_{sn} \) is also given in Table 3. Note, that \( u_s\theta_s/K_{sn} \) should be unity if the NAPL fingers were fully saturated at the finger tip [refer to Eq. (12)]. From the data presented in this figure, is it apparent that, in general, the experimental conditions did not generate NAPL fingers that were fully saturated at their tip. Furthermore, Fig. 7 suggests a trend of slightly increasing NAPL finger saturation with decreasing finger width, and hence increasing \( K_F \). The arithmetic average of the data presented in Fig. 7 is \( (u_s\theta_s/K_{sn})_{av} = 0.84 \pm 0.14 \).

The relationship between unsaturated NAPL finger velocity and saturated NAPL finger velocity is given by

![Graph](image-url)
Fig. 7. $u_n^* / K_{m}$ versus average finger width for all nonaqueous phase liquid infiltration experiments

$$u_n^* = \frac{u_{surf} k_f}{\theta_f} = \frac{k_m k_f}{\theta_f \theta_r},$$  \hspace{1cm} (19)

where the subscript $u=$unsaturated conditions at the finger tip, and $\theta_r=$ the degree of NAPL saturation at the finger tip (which will range from 0 at zero NAPL saturation to 1 at 100% NAPL saturation).

From Eq. (19) and the data presented in Fig. 7, $k_f / \theta_r = 0.84 \pm 0.14$ for the unstable infiltration experiments reported here. An estimate of the degree of NAPL saturation at a finger tip can be obtained from this ratio, if a relationship between $k_f$ and $\theta_r$ is assumed. One of the most widely adopted relationships in unsaturated zone hydrology is the Brooks and Corey relationship (1966)

$$k_f = \left( \frac{\theta_r - \theta_f}{\theta_s - \theta_f} \right)^n$$  \hspace{1cm} (20)

where $\theta_s =$the irreducible fluid content of the medium and $n =$an empirical constant, which has been observed to vary between 3.3 for a silty loam and 4 for uniform glass beads (Brooks and Corey 1964). Both $n$ and $\theta_s$ are properties that can be determined from the matric potential–fluid saturation curve obtained during primary drainage of a soil (Tindall and Kunkel 1999).

To determine $n$ and $\theta_s$ for the 25–40 sand in which the NAPL fingers formed, a water drainage experiment was carried out in the sand (Banno 1996). Note that for a rigid porous medium with negligible fluid–solid surface interactions, both $n$ and $\theta_s$ are properties that are largely dependent on the grain size distribution and packing of a soil, and independent of the interstitial fluid properties and fluid saturation history (Parker et al. 1987). Thus, the parameters $n$ and $\theta_s$ can, theoretically, be determined from a drainage experiment performed using any wetting fluid in a soil. Analysis of the primary drainage curve obtained from the water drainage experiment returned values of $n = 3.7$ and $\theta_s = 0.06$ for the 25–40 sand when packed at a volume porosity $\theta_r = 0.38$.

Substituting $n$, $\theta_r$, and $\theta_s$ into Eq. (20), recognizing that $k_f = 0.84 k_f$, and solving for $\theta_r$ gives $\theta_r = 0.95$. Hence, on average, it is estimated that the observed NAPL fingers were 95% saturated at their tips. The corresponding relative permeability at this degree of saturation is $k_r = 0.8$.

Fig. 8 presents a plot of the average NAPL finger properties as a function of $K_{m}$: (a) $u_n$ versus $K_{m}$; (b) $w_n$ and $\lambda_n$ versus $1/K_{m}$.

**Water Finger Behavior**

In total, four observations of water finger properties were made during Test 10 (Table 3). These observations yielded a mean value of $w / \lambda_1 (= R_w)$ of 0.62 for the water fingers, which is the same as the average observed value of $R_w = 0.62$ for the NAPL fingers.

To determine the water sorptivity $S_w$ of the 25–40 sand, a ponded infiltration test was conducted into a uniformly packed column of the sand. Fig. 9 shows the measured cumulative infiltration of distilled water into the sand as a function of time. Values of $K_m$ and $S_w$ for the sand were determined by fitting the explicit infiltration formula of Barry et al. (1995) to the experimental results (again, see Fig. 9). To perform the fit, $\theta_F$ was set at the volume porosity of the soil column while $h_{surf}$ was set at the ponded surface head of water. The parameter $h_{str}$, which is the
matric potential necessary for a continuous air phase to be established in the soil, was estimated from previous tests results obtained on a similar sand (Culligan et al. 2000). The two undefined values leading to the fit were \( K_{Fw} = 0.047 \text{ cm/s} \) and \( S_w = 0.0052 \text{ cm}^2/\text{s} \).

The value of \( K_{Fw} \) estimated from the infiltration experiment is identical to the value of \( K_{Fw} = 0.047 \text{ cm/s} \) measured from the constant head test (see Table 2). This confirms the fact that conditions at the wetting front are generally saturated during water infiltration into dry soil. In addition, it lends confidence to the method of analysis that was used to determine \( S_w \).

To verify that the finger properties for water infiltration into a dry soil can be reasonably estimated from a knowledge of \( S_w \), \( K_{Fw} \), and \( \theta_g \), Eqs. (11) and (12) were used to predict \( w_w \) and \( u_w \) for the conditions of Test 10. From Eq. (11), the estimated value of \( w_w \) is 3.1 cm, which is in good agreement with the averaged observed value of 2.9 cm (Table 3). From Eq. (12), the estimated value of \( u_w \) is 0.127 cm/s, which is in excellent agreement with the averaged observed value of 0.126 cm/s.

Because Eqs. (11) and (12) provide reasonable estimates of the observed values of \( w_w \) and \( u_w \), it is possible to conclude that, unlike the NAPL fingers, the water fingers were 100% saturated at their tips.

**Investigation of Scaling Analysis**

To determine if scaling analysis can be used to predict the properties of NAPL fingers in a uniform soil, Eqs. (16) and (17) were used in conjunction with the fluid properties listed in Table 1, to estimate \( w_n \) and \( u_n \). In both formulas, the predicted values of \( w_w \) and \( u_w \) were adopted (see above). In addition, the contact angles for Soltrol and water in the 25–40 sand were taken as the contact angles of these fluids on polished quartz, respectively. Note that because observed values of \( w/w \) were used to infer \( R \), the validity of Eq. (18) could not be independently investigated.

Fig. 9. Results of ponded water infiltration experiment into a 72-mm diameter, 220 mm high column of initially dry 25–40 sand. Cumulative infiltration is cumulative volume of infiltration per cross-sectional area of soil. Cumulative volume of infiltration was obtained by directly measuring volume of water required to maintain 1.5 cm head of ponded water on soil surface. Experimental data, shown as symbols, are fit using explicit infiltration formula of Barry et al. (1995). Parameters leading to the fit are: \( O_p = 0.34 \), \( k_{surf} = 1.5 \text{ cm} \), \( h_{surf} = -0.2 \text{ cm} \), \( K_{Fw} = 0.047 \text{ cm/s} \), and \( S_w = 0.0052 \text{ cm}^2/\text{s} \).

Fig. 10 presents a plot of observed NAPL finger width versus predicted NAPL finger width using Eq. (16). From the plot, it is noted that Eq. (16) underpredicts the average observed finger width. Some disagreement between Eq. (16) and the experimental data is expected, as the scaling analysis was developed assuming that both the NAPL and the water fingers were saturated. In the case of the water fingers, this hypothesis was found to be reasonable. However, in the case of the LNAFL fingers, finger-tip saturations of less than 100% were observed. A linear regression of the data predicted using Eq. (16) yields the relationship \( \text{W}_{w\text{predicted}} = 0.47 \text{W}_{w\text{observed}} \); \( R^2 = 0.95 \).

Fig. 10. Predicted nonaqueous phase liquid finger width versus observed nonaqueous phase liquid finger width.
Fig. 11. Predicted nonaqueous phase liquid finger velocity versus observed nonaqueous phase liquid finger velocity.

Fig. 11 presents a plot of observed NAPL finger velocity versus predicted NAPL finger velocity using Eq. (17). In this case, \( u_n \) is overpredicted by Eq. (17). Again, this can be attributed to the fact that the NAPL fingers were not fully saturated at their tips. A linear regression of the data predicted using Eq. (17) returns the relationship \( u_{\text{un(predicted)}} = 1.14u_{\text{un(observed)}} \); \( R^2 = 0.94 \).

The partial saturation of NAPL fingers in the scaling analysis can easily be accounted for by recognizing that \( w_{\text{un}} = \frac{w_{\text{sw}}}{k_r \theta_r} \) [see Eq. (11)] and \( u_{\text{un}} = \frac{u_{\text{sw}}}{k_r \theta_r} \) [see Eq. (20)]. Hence, the generalized form of Eqs. (16) and (17) becomes

\[
\frac{w_{\text{un}}}{w_{\text{sw}}} = \frac{\sigma_n \cos \phi_n \rho_w g_w (1 - R_w)}{\sigma_w \cos \phi_w \rho_n g_n (1 - R_n)} \frac{k_r}{k_w} \theta_r
\]

(21)

and

\[
u_{\text{un}} = \frac{\rho_n u_{\text{sw}} k_r}{\rho_w u_{\text{sw}} g_w \theta_r} \theta_r
\]

(22)

Thus, the effect of a reducing NAPL saturation at the NAPL finger tip is to increase the NAPL finger width, and reduce the NAPL finger velocity. Note that Eqs. (21) and (22) have been derived assuming that the hydraulic gradient at the NAPL finger tip and the infiltrating fluid sorptivity are not greatly affected by the degree of saturation at the infiltration front.

The predictions given by Eqs. (21) and (22) are also plotted on Figs. 11 and 12, respectively. In both cases, accounting for the partial saturation of NAPL at the finger tip improves the accuracy of the predictions. From Fig. 11, a linear regression of the data predicted using Eq. (21) returns \( w_{\text{un(predicted)}} = 0.67w_{\text{un(observed)}} ; R^2 = 0.95 \). From Fig. 12, the agreement between Eq. (22) and the observed finger velocity is described by \( u_{\text{un(predicted)}} = 0.91u_{\text{un(observed)}} ; R^2 = 0.94 \).

It is noted that the predictions of the NAPL finger widths are less accurate than the predictions of the NAPL finger velocities. From Eqs. (16) and (21) it can be seen that the predictions for NAPL finger widths are very sensitive to the ratio \( \sigma_n \cos \phi_n / \sigma_w \cos \phi_w \), which was calculated using the pure properties of each fluid measured in a clean system. Although the surface tension and wetting properties of the aliphatic oil, SOLTROL 220, are not changed readily by the presence of contaminants, the surface tension and wetting properties of water can be (Brooks and Corey 1966). In an investigation into the effects of contamination on liquid properties, Holguin (1999) demonstrated an average 25% reduction in \( \sigma \_w \cos \phi_w \) in 2.6-mm-diameter glass capillary tubes that had been rinsed with tap water, rather than triple rinsed with acetonite, methanol, and distilled water. For SOLTROL 220, Holguin reported a corresponding reduction in \( \sigma_n \cos \phi_n \) of only 7%. Thus, the presence of even minor contamination in a system can increase the ratio \( \sigma_n \cos \phi_n / \sigma_w \cos \phi_w \) by a measurable amount. For the experiments reported here, any increase in the ratio \( \sigma_n \cos \phi_n / \sigma_w \cos \phi_w \), which might be justified due to the nonsterile nature of the experimental environment, would improve the accuracy of predictions for \( w_{un} \). For example, if, as per Holguin’s observations, contaminants present in the experimental environment reduced \( \sigma_n \cos \phi_n \) by 25% and \( \sigma_w \cos \phi_w \) by 7%, then \( w_{un(predicted)} \) would equal 0.83 \( w_{un(observed)} \); \( R^2 = 0.95 \).

Conclusions

The technique of geotechnical centrifuge testing was used to examine two-dimensional NAPL finger behavior in uniform, dry sand under a range of different infiltration conditions. In total, the properties of 87 NAPL fingers were observed. The observed finger tip velocities ranged from 0.01 to 0.3 cm/s, while the observed finger widths ranged from 0.3 to 3.6 cm. From the experimental data, it was concluded that, asymptotically, the NAPL fingers were not fully saturated. Instead, it was estimated that, typically, the fingers were only 95% saturated at their tips.

For comparison, the behavior of water fingers was also examined using the same experimental setup. In contrast, and in agreement with previous work reported in the literature (e.g., Glass et al. 1989), it was found that, asymptotically, the water fingers were saturated at their tips. In addition, it was found that the water finger properties could be predicted using known porous medium and fluid properties.

Based on the assumption that both finger types were saturated, a scaling analysis was developed that related the properties of NAPL fingers to the properties of water fingers formed in the same soil. Predictions developed for water fingers, in conjunction with this analysis, were used to estimate the NAPL finger properties. A comparison between the estimated and observed NAPL finger properties demonstrated that the ensuing model underpredicted the NAPL finger width, while it overpredicted the NAPL finger velocity. Nonetheless, based simply on a knowledge of the NAPL fluid properties and the soil water properties \( K_w, S_{fw}, \) and \( \theta_s \), the model was able to predict NAPL finger widths to within approximately 50%, and NAPL finger velocities to within approximately 15%. As a first estimate for the purpose of scoping a preliminary investigation at a NAPL contaminated site, this degree of agreement is not unreasonable.

An adjustment of the scaling analysis was made to account for the fact that the NAPL fingers were not fully saturated. Predictions made using the adjusted analysis were able to forecast NAPL finger widths to within approximately 30%, and NAPL finger velocities to within 10%. Although a priori knowledge of NAPL finger saturations will not generally be known at a waste site, small-scale controlled field, and/or laboratory experiments, or data gathered from other, similar, sites, might be used to surmise this information.

Predictions of NAPL finger behavior in instability-prone situations are required in the development of strategies for preventing and cleaning up groundwater pollution. The work described
above has demonstrated that measured soil and fluid properties can be used to obtain a sensible first estimate of the properties of NAPL fingers formed in predominately dry soil. If knowledge of the likely saturation conditions at the NAPL finger tips can be acquired from small scale testing or field observation, then the accuracy of this estimate can be improved.

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Notation

The following symbols are used in this paper:

\[ d = \text{finger diameter}; \]
\[ d_{so} = \text{average soil particle size}; \]
\[ g = \text{acceleration due to gravity}; \]
\[ h = \text{matric potential}; \]
\[ h_{surf} = \text{surface level of water during ponded infiltration test}; \]
\[ h_{str} = \text{matric potential required to establish continuous air–phase in soil during ponded infiltration test}; \]
\[ K_F = \text{hydraulic conductivity at infiltration front}; \]
\[ k = \text{intrinsic permeability of soil}; \]
\[ k_F = \text{permeability at infiltration front}; \]
\[ k_r = \text{relative permeability}; \]
\[ N = \text{scaling factor (g level) for centrifuge experiments}; \]
\[ n = \text{Brooks–Corey empirical constant}; \]
\[ q_F = \text{average fluid flux through finger}; \]
\[ q_i = \text{infiltrating fluid flux}; \]
\[ R = \text{system flux ratio (q_i/K_F)}; \]
\[ r = \text{radial distance of experiment from central axis of geotechnical centrifuge}; \]
\[ S = \text{fluid sorptivity}; \]
\[ s = \text{intrinsic sorptivity}; \]
\[ u = \text{finger velocity}; \]
\[ w = \text{finger width}; \]
\[ z = \text{length of finger}; \]
\[ \theta_F = \text{average volumetric water content of finger}; \]
\[ \theta_i = \text{irreducible fluid content of soil}; \]
\[ \theta_0 = \text{initial volumetric water content of soil}; \]
\[ \theta_r = \text{degree of saturation}; \]
\[ \theta_s = \text{saturated volumetric water content of soil}; \]
\[ \lambda = \text{finger spacing}; \]
\[ \mu = \text{fluid viscosity}; \]
\[ \rho = \text{fluid density}; \]
\[ \sigma = \text{surface tension of fluid}; \]
\[ \phi = \text{contact angle}; \]
\[ \omega = \text{rotational speed of geotechnical centrifuge}; \]

subscripts

\[ n = \text{NAPL as infiltrating fluid}; \]
\[ s = \text{saturated property}; \]
\[ u = \text{unsaturated property}; \]
\[ w = \text{water as infiltrating fluid}; \]

References