Wetting Front Instability in Homogeneous Sandy Soils under Continuous Infiltration

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ABSTRACT

The boundary conditions required to produce unstable wetting have not been thoroughly investigated. Our experiments demonstrate that wetting front instability occurs in two- and three-dimensional homogeneous columns of sandy soils under continuous nonponding infiltration, simulating rain-fed conditions. Instabilities were visualized in two-dimensional experiments by observing the light transmitted through partially saturated silica sand, and by dye staining in a three-dimensional experiment. The dimensions of the instabilities observed corresponded well to the previous results found in studies of layered soils, indicating that the theories developed previously apply to rain-fed infiltration without layering. Our study extends the range of situations where fingered flow is predicted to occur, which indicates that models used to calculate contaminant transport and loading in uniform coarse soils should include the effects of fingered flow.

Two types of experiments have been carried out to date to investigate unstable flow behavior. The first experimental setup employed is a two-layer configuration, where a fluid is ponded over an upper layer of fine sand, which limits the flux entering a more permeable coarse sand (Hill and Parlange, 1972; White et al., 1976, 1977; Diment and Watson, 1985; Glass et al., 1988, 1989a,c; Baker and Hillel, 1990). The flux entering the lower layer is less than the saturated hydraulic conductivity of the coarse soil, where the front becomes unstable and forms columns of flow (fingers).

The second experimental method is based on redistribution of water within a porous medium. In this case, a viscous liquid is ponded on the surface of a homogeneous medium; after the wetting front reaches a prescribed depth, application is suspended with the fluid continuing to redistribute itself under unsaturated conditions. Such flow has been observed to become unstable immediately after cessation of ponding (White et al., 1976, 1977; Diment and Watson, 1985; Tamai et al., 1987). Redistribution experiments are limited to discontinuous flow conditions.

Various predictions have been made regarding the stability of wetting fronts in homogeneous soils under continuous nonponding infiltration. Raats (1973) predicted that nonponding infiltration should be unstable in all cases, while Philip (1975) predicted that fingering would not occur in a homogeneous wetting medium under such infiltration. Indicative of the widespread belief that nonponding infiltration into homogeneous soils is stable, Diment et al. (1982) stated in the conclusion of their theoretical analysis “the [preceding] analysis will be applied to soil water systems such as infiltration into homogeneous profiles (these are known to be stable) ... “. Although primarily interested in layered soils, Hillel and Baker (1988) concur with Raats (1973). Baker and Hillel (1990) showed experimentally that their predicted fraction of wetted soil (Hillel and Baker, 1988) was accurate, but employed a two-layer setup, leaving open the issue of stability in homogeneous soils. Most recently, Hendrickx and Dekker (1991) reported evidence of unstable wetting in homogeneous soils under natural rain-fed conditions.

Given the broad occurrence of infiltration under unsaturated conditions, experimental investigation of instability in homogeneous coarse soils at fluxes below saturated hydraulic conductivity merit further investigation. The objectives of this study were to: (i) examine the stability of wetting fronts arising from rainfall events on coarse-grained soils at infiltration rates less than the saturated hydraulic conductivity, and (ii) compare the instabilities observed in these conditions with fingered flow observed in two-layer soil configurations.

ESTIMATION OF FINGER PARAMETERS

To describe the effect of fingered flow on groundwater quality, the width and velocity of fingers need be known as functions of the soil and infiltration conditions. Finger diameters predicted by Parlange and Hill (1976) for two-dimensional systems, which were derived assuming the existence of a saturated region above the region of fingered flow, corresponded well to the experimental results for two-dimensional two-layer systems reported by Glass et al. (1989c). Parlange and Hill (1976) also report observing finger velocity close to $K_s(a - \theta_c)$, where $K_s$ is the saturated hydraulic conductivity, $\theta_c$ is the saturated water content, and $a$ is the initial water content of the medium. Glass et al. (1989b) fit the expression presented by Parlange and Hill to experimental observations using dimensional analysis, obtaining finger velocity as $K_s[(\theta_a - \theta_c)(0.1 + 0.9q_f/K_s)]$, which differs only slightly from the earlier result since the finger flux, $q_f$, is generally close to $K_s$. Glass et al. (1990) derived relationships for three-dimensional finger width using the approach that was successfully employed by Parlange and Hill (1976) for two-dimensional fingers. Three-dimensional fingers were predicted to have diameters of 4.8/π times that of two-dimensional fingers, which was supported well by their experimental findings. The dimension of a finger refers to the character of the tip. In two-dimensional experiments, the fingers are confined to having curvature in a plane, whereas fingers growing in three-dimensional experiments have tip curvature in two orthogonal planes.

The experiments that are described below may be employed to check if the theories of Parlange and Hill (1976) and Glass et al. (1990), which were tested primarily in ponded two-layer configurations, are ap-

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applicable to homogeneous soils with continuous infiltration at rates less than saturated hydraulic conductivity.

METHODS

Two- and three-dimensional column studies were carried out under uniform continuous infiltration. Data were collected in a continuous, nondestructive manner in the two-dimensional experiments. The destructive dye tracer method was employed in the three-dimensional experiments to illustrate features of the process that play an important role in applications of fingered flow in natural settings, and support the generality of the two-dimensional system results suggested by Glass et al. (1990).

Two-Dimensional Experiments

In these experiments, water was applied to the surface of a uniform, homogeneous, vertical column of gravity-packed silica sand. The sand material was graded by collecting the fraction of silica sand passing through standard sieves. Sand particles between 0.59- and 0.85-mm diameter (U.S. standard sieves no. 30 and 20) referred to as 20-30 sand, were used for the experiment. The sand was washed in a dilute solution of laboratory glassware soap, followed by five rinses with tap water and two rinses with distilled water. Each rinse was accompanied by a 5-min air agitation. A 1-cm-thick, 5.14-cm-wide, 40-cm-high column of sand was held in place by 1-cm-thick glass sheets at each side, and was open to the atmosphere at the top surface. The column was packed by pouring the sand through a 55-cm-wide funnel fitted with a sequence of three 3-mm-opening steel meshes on approximately 10-cm vertical spacing. The porosity of the packed column was 0.40, as determined by weighing the fully loaded, dry chamber assuming a silica density of 2.64 g/cm³. A uniform flux to the soil surface, simulating rain-fed infiltration, was established by misting water from a chain-driven shuttle uniformly over the upper surface of the sample. The column was irrigated with 0.862 cm³/s over the 51.4 cm² surface of the experiment (1.68 × 10⁻⁴ m/s). The development of the flow field was recorded on a videotape.

Three-Dimensional Experiments

The form for the column was constructed of 10-cm-high rings of 29.7-cm (12-inch) i.d. polyvinyl chloride (PVC) pipe. One-millimeter-thick spacers were placed between rings to allow displaced air to escape during infiltration. In total, four rings were used to produce a full column height of 40 cm. As in the packing of the two-dimensional experiment, the sand was fed through a three-layer steel mesh randomizer. In this experiment, 14-20 grade sands were used (0.85-1.4 mm). In the case of the two-dimensional experiment, the coarser sand would have given rise to fingers that were approximately the width of the chamber, making them strongly three dimensional (in the sense of being radially symmetric rather than planar). In the three-dimensional experiment, the larger fingers arising from the use of finer sand would have caused even more merger, clouding observation of the fingering process, and making it more difficult to measure finger diameter. In addition, the use of two sands allowed testing of the scaling theory proposed by Parlange et al. (1990).

After filling, a 20-kg bag of sand was placed on top of the sand, and the column was dropped 200 times from an elevation of 5 cm to consolidate the media, eliminating lower bulk densification that may have resulted from electrostatic repulsion evident during filling. The gaps created by the 1-mm spacer were sealed with adhesive tape to avoid sand loss during packing. The top 5 cm of the column of sand was removed to eliminate surface nonuniformities in bulk density generated during packing. Packing of the three-dimensional chamber was carried out to overcome the effect of electrostatic buildup that occurred in the sand as it entered the plastic column, which was not an issue in the two-dimensional setup. A bidirectional rainfall simulator was employed that defined a plane of uniform irrigation (following Andreini and Steenhuis, 1990). The application rate of 1.50 cm³/s (2.12 × 10⁻⁴ m/s) was found to be spatially uniform, with a coefficient of variation of 3.7% among 16 collection bottles placed over the experimental area.

In the final 7 min of the experiment, distilled water with 0.1% FD&C blue dye no. 1 was substituted for the irrigation supply. Undisturbed 10-cm-high rings of sand from the experiment were obtained by sliding steel sheets into the column between the PVC rings. The dye stains clearly delineated the areas of flow, which were recorded by tracing the boundary of the dyed area onto transparent plastic sheets. Each 10-cm-thick section of the column was then frozen. When the PVC ring was removed from the frozen section, the loose dry sand was removed, revealing the frozen wetted sections in three dimensions.

RESULTS

Figure 1 illustrates the results of infiltration in a two-dimensional experiment. The spacing of the fingers was very regular, with each finger carrying approximately an equal fraction of the flux. This experiment was repeated 14 times in freshly packed sand. The wetted pattern was similar in the depth of the horizontal wetted area, the number of fingers, and finger spacing, with no evidence of systematic positioning of fingers due to packing or irrigation. Finger dimensions are listed in Table 1 for the experiment shown in Fig. 1.

In the three-dimensional experiment, the correspondence between finger dimensions and dye location was verified by observing that, when frozen, no blue sand fell away from fingers, while the surfaces of the frozen fingers were strongly dyed. Considering successive 10-cm layers in the column (Fig. 2), the number of fingers were 43 at 10-cm depth, 37 at 20 cm, and

Fig. 1. The formation of wetting front instability in a two-dimensional 0.59-0.85-mm sand experiment (51.4 cm wide by 40 cm high). Numbers indicate the time (in seconds) at which the wetting front reached the position indicated by lines drawn across fingers.
Table 1. The velocities and widths of the five fingers shown in Fig. 1. The consistency of the velocity is verified by calculating the $r^2$ value of the fit between a straight line and the time vs. displacement data for each finger.

<table>
<thead>
<tr>
<th>Finger number (left to right)</th>
<th>Velocity</th>
<th>$r^2$†</th>
<th>Width</th>
<th>SD‡</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.314</td>
<td>0.998</td>
<td>1.51</td>
<td>0.25</td>
</tr>
<tr>
<td>2</td>
<td>0.369</td>
<td>0.999</td>
<td>1.60</td>
<td>0.31</td>
</tr>
<tr>
<td>3</td>
<td>0.441</td>
<td>0.999</td>
<td>1.88</td>
<td>0.23</td>
</tr>
<tr>
<td>4</td>
<td>0.381</td>
<td>0.994</td>
<td>1.74</td>
<td>0.25</td>
</tr>
<tr>
<td>5</td>
<td>0.404</td>
<td>0.997</td>
<td>1.73</td>
<td>0.26</td>
</tr>
<tr>
<td>Average</td>
<td>0.382</td>
<td>0.997</td>
<td>1.69</td>
<td>0.26</td>
</tr>
</tbody>
</table>

† Coefficient of determination for fit of growth in length to constant velocity.
‡ Standard deviation of width of finger.

Table 2. Results of three-dimensional homogeneous sand study, giving the number and diameter of fingers ($d_m$) at depths of 10, 20, and 30 cm. The standard deviation (SD) in finger diameter and expected finger width ($d_{mac}$) are also reported.

<table>
<thead>
<tr>
<th>Layer</th>
<th>Number of fingers</th>
<th>$d_m$</th>
<th>$d_{mac}$</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>43</td>
<td>1.89</td>
<td>2.28</td>
<td>0.96</td>
</tr>
<tr>
<td>2</td>
<td>37</td>
<td>1.78</td>
<td>2.28</td>
<td>0.99</td>
</tr>
<tr>
<td>3</td>
<td>28</td>
<td>1.76</td>
<td>2.28</td>
<td>0.97</td>
</tr>
</tbody>
</table>

Fig. 2. The position of fingers as indicated by dye (A) 10 cm, (B) 20 cm, and (C) 30 cm below the irrigated surface of the three-dimensional column experiment.

28 fingers at the depth of 30 cm. On inspection of the frozen samples, it was apparent that the decrease in finger count was the result of finger merger. Finger diameters were measured from Fig. 2, and are listed in Table 2. Finger area was measured by digitizing the tracings and determining the area of each finger using the public domain Image 1.16 software (Rasband, 1989).

DISCUSSION

We may compare the width and velocity data from the two-dimensional experiment with the results obtained by Glass et al. (1989c), who employed a two-layer experimental setup. The equations derived by Glass et al. (1989c) require measurement of sorptivity, saturated hydraulic conductivity, porosity (to estimate saturated water content), flux through the finger, and cross-sectional area of the finger. Measurement of saturated hydraulic conductivity was carried out using a constant-head cylindrical 5-cm-i.d. 20-cm-high up-flow column. In three repeated measurements, $K_s$ was found to be $3.4 \times 10^{-3}$ m/s with repeatability of $\pm 0.3 \times 10^{-3}$ m/s. The average value of flux in the fingered-flow section of the chamber (applied flux divided by wetted area) was $1.02 \times 10^{-3}$ m/s. The predicted average finger growth velocity for the two-dimensional studies was $3.14 \times 10^{-3}$ m/s, quite close to the measured value of $3.82 \times 10^{-3}$ m/s (Table 1).

Predicting the width of the fingers required the value of the soil sorptivity which is quite difficult to measure for coarse soils. The sorptivity, however, may be obtained by scaling the result of Glass et al. (1989c) using Eq. [16] of Glass et al. (1989b), based on the Miller and Miller (1956) scaling theory. Miller and Miller (1956) scaling has been shown to apply well to prediction of finger dimension (Parlangie et al., 1990; Selker et al., 1991). Scaling is appropriate in this case since the sands used in this experiment and those of Glass et al. (1989b,c) are from the same source, with bulk density of packing within 2% by volume, and of similar grain-size distribution (i.e., similar geometrically as required for Miller and Miller scaling). Glass et al. (1990) report a sorptivity of $9.17 \times 10^{-6}$ m²/s for 14-20 sand with an average porosity of 0.42. For the 20-30 sand employed here, the estimate of sorptivity is then $5.9 \times 10^{-6}$ m²/s. The uncertainty in the value given by Glass et al. (1990) is quite high due to the difficulty in measuring sorptivity in coarse materials. If the water entry pressure employed by Glass et al. (1989c) is accurate to $\pm 10\%$ (in keeping with our experimental findings of actual measurements of the pressure in the tips of growing instabilities [Selker 1991]), and all other parameters are known exactly, an uncertainty in sorptivity of $\pm 6 \times 10^{-6}$ m²/s arises. Clearly, estimates of finger dimensions based on this
value of sorptivity are only useful for order of magnitude level comparisons. From these values, predicted finger width is 2 cm, which is well within the uncertainty in the calculation of the values listed in Table 1.

Turning to the three-dimensional experiments, Glass et al. (1990) report the saturated hydraulic conductivity of 14–20 sand to be $4.12 \times 10^{-3}$ m/s. The flux ratio, $R_t$, in our experiment (defined as the system flux divided by the saturated hydraulic conductivity) is on the order of 0.01. The dependence of finger diameter on $R_t$ is $d = d_0[(1 - R_t)]^{1/2}$, where $d_0$ is the lower bound on finger diameter (Glass et al., 1990). Setting $R_t = 0$ for this case does not result in loss of accuracy. These parameters provide a predicted finger diameter of 2.28 cm. As was the case for two-dimensional fingers, the results of Glass et al. (1990) correspond well with the results of these experiments (Table 2).

The expected diameter of fingers in three dimensions are $4.8/\pi$ times those of two-dimensional fingers for a given medium (Glass et al., 1990). Using this ratio, with the scaling results of Parlange et al. (1990) as supported by Baker and Hillel (1990) and Selker et al. (1991), we may compare the two-dimensional finger width observed in 20–30 sand to the diameter of three-dimensional fingers obtained in 14–20 sand. Calculating from the two-dimensional width, the predicted width of the three-dimensional fingers is given by Parlange et al. (1990) as

$$d_{3d} = d_{2d} \frac{4.8}{\pi} \frac{M_{2d}}{M_{3d}} \quad [1]$$

where $M_{2d}$ and $M_{3d}$ are the characteristic particle sizes of sand in the two- and three-dimensional experiments. Given the very limited particle-size distribution of the sands used in this study, we simply employed the average of the smallest and largest particles of each sand class, yielding 0.72 and 1.13 mm for $M_{2d}$ and $M_{3d}$, respectively. Using the average finger width of 1.69 cm for two-dimensional fingers from Table 1, the predicted three-dimensional finger diameter is 1.65 cm in 14–20 sand, which is within 10% of the average diameter value reported in Table 2.

CONCLUSIONS

These experimental results demonstrate that fingering can be a prominent feature of flow through homogeneous soil systems under continuous nonpounding infiltration. Rather than being restricted to either situations that have fine sand over coarse (layered soils) or only where rainfall is characterized by a pulse of saturated infiltration (redistribution), it is now apparent that fingered flow can arise in a wider range of unstratified sandy soils that experience unsaturated flow. Given recent findings of finger persistence (Glass et al., 1989d), it may be that the soils need only be dry at one time to give rise to fingers, which could then persist for very long periods of time. Further research is required to define more precisely the conditions under which fingering should be expected based on the conditions of climate, vegetation, and soil.

The dimensions of fingers observed are of the same scale as those produced by Glass et al. (1989c) and agree with the analytical formulations derived therein to within the uncertainty in the calculations. The merger of fingers that was noted in the three-dimensional experiment reduced the number of fingers and wetted area by 35% through a depth of 30 cm. Finger spacing should be expected to vary considerably under field conditions where anisotropy, impurities, and soil structure (Steenhuis et al., 1990) will tend to reduce the average finger width, increase finger growth velocity, and increase the space between fingers through merger and funnel flow (Steenhuis et al., 1990). This conclusion is supported by the findings of Kung (1990), where a uniformly irrigated 33-m² area drained into one column of flow before reaching the water table at 6 m.

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REFERENCES


